MATHEMATICS

KPK BOARD NOTES

9TH CLASS

Presented by:

Urdu Books Whatsapp Group

STUDY GROUP

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UNIT 2:
REAL AND COMPLEX NUMBERS

Real Numbers:

Definition: The union or sum of rational and irrational numbers is known as *real numbers*. It is denoted by R and written as $R = Q \cup Q'$

Rational Number:

That number which can be written in the form of p/q where $p, q \in z$ and $q \neq 0$ i.e.

$Q = \left\{ x / x = p/q ; p, q \in z, q \neq 0 \right\}$ is called rational number.

Here Q means quotient which means ratio. Every rational number is written either as terminating decimal or a repeating decimal. For example;

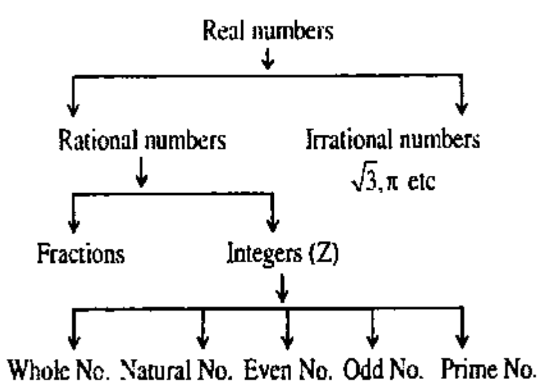
$\frac{3}{8} = 0.375,\ \frac{1}{3} = 0.333....... = 0.\overline{3}$

Here the bar $(0.\overline{3})$ means that digit 3 is repeated infinite times.

Irrational Number:

Definition: That number which is written as non-terminating and non-repeated decimal is known as *irrational number*. It is denoted by Q'. For example;

$\sqrt{3}, \pi, ...1.2345....$ etc.



EXAMPLE 1

Write $\frac{3}{8}$ and $\frac{2}{15}$ as a decimal fractions.

Solution:

```
   0.375  ← This is a terminating decimal
8)3.000
  -24
   60
  -56
   40
  -40
    0     ← The remainder is zero.
```

$\frac{3}{8} = 0.375$

```
    0.133  ← This is a non-terminating decimal
15)2.000
   -15
    50
   -45
     5     ← The remainder is never zero.
```

$\frac{2}{15} = 0.133...... = 0.1\overline{3}$

An irrational number is a number whose decimal representation never terminates or repeats.

For example

$0.010010001000001....,\ \sqrt{7} = 2.6457513...,$

And $\pi = 3.1415927...$

The real number system is summarized in the figure given below.

EXERCISE 2.1

Q1: In questions 1-10 consider the numbers 2.5, 3, $5/7$, -1.96, $\sqrt{36}$, $-7/6$, $\sqrt{3}$, -9, 1, $\sqrt{7}$, $-\sqrt{14}$, π, $4\frac{2}{3}$, 0.333.....

1. Which are whole numbers?

Ans. Whole numbers are: $3, 1, \sqrt{36}$

2. Which are integers?

Ans. Integers are: $3, 1, -9, \sqrt{36}$

3. Which are irrational numbers?

Ans. Irrational numbers are: $\sqrt{3}, \sqrt{7}, -\sqrt{14}, \pi$

4. Which are natural numbers?

Ans. Natural numbers are: $3, \sqrt{36}, 1$

5. Which are rational numbers?

Ans. Rational numbers are:

$2.5, 3, \frac{5}{7}, -1.96, \sqrt{36}, \frac{-7}{6}, -9, 1, 4\frac{2}{3}$,

$0.333....$ These are rational numbers.

6. Which are real numbers?

Ans. All given numbers are real.

7. Which are rational numbers but not integers?

Ans. The following numbers are rational numbers but not integers.

$2.5, \frac{5}{7}, -1.96, \frac{-7}{6}, 4\frac{2}{3}, 0.333 ...$

8. Which are integers but not whole numbers?

Ans. -9

9. Which are integers but not natural numbers?

Ans. -9

This is integer but no whole number because -9 is integer but not natural number.

10. Which are real numbers but not integers?

Ans. $2.5, \frac{5}{7}, -1.96, \frac{-7}{6}, \sqrt{3}, \sqrt{7}, -\sqrt{14}, \pi,$

$4\frac{2}{3}, 0.333....$

These all are real numbers but not integers.

11. Write the decimal representation of the following numbers:

$\frac{1}{6}, \frac{6}{7}, \frac{2}{9}, \frac{1}{8}$

Solution:

The decimal form of $\frac{1}{6} = 0.1666.........$

OR $\frac{1}{6} = 0.1\overline{6}$

The decimal form of $\frac{6}{7} = 0.8571428571$

The decimal form of $\frac{2}{9} = 0.2222......$

OR $\frac{2}{9} = 0.\overline{2}$

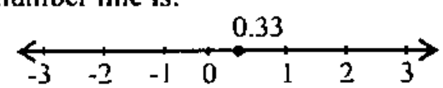
The decimal form of $\frac{1}{8} = 0.125$

12. Depict each number on a number line. $\frac{1}{3}, \frac{1}{4}, \frac{1}{9}, \frac{1}{10}$

Solution:

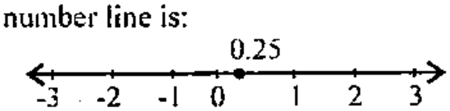
Given $\frac{1}{3} = 0.33$

Its number line is:



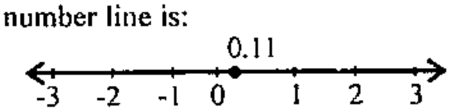
$\frac{1}{4} = 0.25$

Its number line is:



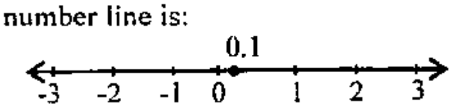
$\frac{1}{9} = 0.11$

Its number line is:



$\frac{1}{10} = 0.1$

Its number line is:



Properties of Real Numbers:

a) Addition Properties:

(i) Closure Property w.r.t Addition: The sum of two real numbers is again real number. i.e. For $a, b \in R \Rightarrow a + b \in R$

(ii) Associative Property w.r.t Addition: For $a, b, c \in R \Rightarrow a + (b + c) = (a + b) + c$

(iii) Additive Identity: If $a \in R$ there exists $0 \in R$ such that $a + 0 = 0 + a = a$ then 0 is called additive identity element.

ختم نبوت ﷺ زندہ باد **عظمت صحابہ زندہ باد**

السلام علیکم ورحمۃ اللہ وبرکاتہ:

معزز ممبران: آپ کا وٹس ایپ گروپ ایڈمن "**اردو بکس**" آپ سے مخاطب ہے۔

آپ تمام ممبران سے گزارش ہے کہ:

- ❖ گروپ میں صرف PDF کتب پوسٹ کی جاتی ہیں لہذا کتب کے متعلق اپنے کمنٹس / ریویوز ضرور دیں۔ گروپ میں بغیر ایڈمن کی اجازت کے کسی بھی قسم کی (اسلامی و غیر اسلامی، اخلاقی، تحریری) پوسٹ کرنا سختی سے منع ہے۔
- ❖ گروپ میں معزز، پڑھے لکھے، سلجھے ہوئے ممبرز موجود ہیں اخلاقیات کی پابندی کریں اور گروپ رولز کو فالو کریں بصورت دیگر معزز ممبرز کی بہتری کی خاطر ریموو کر دیا جائے گا۔
- ❖ کوئی بھی ممبر کسی بھی ممبر کو انباکس میں میسج، مس کال، کال نہیں کرے گا۔ رپورٹ پر فوری ریموو کر کے کاروائی عمل میں لائے گی۔
- ❖ ہمارے کسی بھی گروپ میں سیاسی و فرقہ واریت کی بحث کی قطعاً کوئی گنجائش نہیں ہے۔
- ❖ اگر کسی کو بھی گروپ کے متعلق کسی قسم کی شکایت یا تجویز کی صورت میں ایڈمن سے رابطہ کیجئے۔
- ❖ سب سے اہم بات:

گروپ میں کسی بھی قادیانی، مرزائی، احمدی، گستاخِ رسول، گستاخِ امہات المؤمنین، گستاخِ صحابہ و خلفائے راشدین حضرت ابو بکر صدیق، حضرت عمر فاروق، حضرت عثمان غنی، حضرت علی المرتضیٰ، حضرت حسنین کریمین رضوان اللہ تعالیٰ اجمعین، گستاخِ اہلبیت یا ایسے غیر مسلم جو اسلام اور پاکستان کے خلاف پراپیگنڈا میں مصروف ہیں یا ان کے روحانی و ذہنی سپورٹرز کے لئے کوئی گنجائش نہیں ہے لہذا ایسے اشخاص بالکل بھی گروپ جوائن کرنے کی زحمت نہ کریں۔ معلوم ہونے پر فوراً ریموو کر دیا جائے گا۔

- ❖ تمام کتب انٹرنیٹ سے تلاش / ڈاؤنلوڈ کر کے **فری آف کاسٹ** وٹس ایپ گروپ میں شیئر کی جاتی ہیں۔ جو کتاب نہیں ملتی اس کے لئے معذرت کر لی جاتی ہے۔ جس میں محنت بھی صَرف ہوتی ہے لیکن ہمیں آپ سے صرف دعاؤں کی درخواست ہے۔
- ❖ عمران سیریز کے شوقین کیلئے علیحدہ سے عمران سیریز گروپ موجود ہے۔
- ❖ **لیڈیز کے لئے الگ گروپ کی سہولت موجود ہے جس کے لئے ویریفکیشن ضروری ہے۔**
- ❖ اردو کتب / عمران سیریز یا سٹڈی گروپ میں ایڈ ہونے کے لئے ایڈمن سے وٹس ایپ پر بذریعہ میسج رابطہ کریں اور جواب کا انتظار فرمائیں۔ برائے مہربانی اخلاقیات کا خیال رکھتے ہوئے موبائل پر کال یا ایم ایس کرنے کی کوشش ہرگز نہ کریں۔ ورنہ گروپس سے تو ریموو کیا ہی جائے گا بلاک بھی کیا جائے گا۔

نوٹ: ہمارے کسی گروپ کی کوئی فیس نہیں ہے۔ سب فی سبیل اللہ ہے

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اللہ تبارک تعالیٰ ہم سب کا حامی و ناصر ہو

(iv) Additive Inverse: For $a \in R$ there exists $a' \in R$ such that $a + a' = a' + a = 0$ then a' is called additive inverse of a and it is denoted by $-a$.

Example: $2 + (-2) = 0$ and $3 + (-3) = 0$

(v) Commutative Property w.r.t Addition: For $a, b \in R \Rightarrow a + b = b + a$

b) Multiplication Properties:

(i) Closure Property w.r.t Multiplication: For $a, b \in R \Rightarrow a.b \in R$ i.e. the product of two real numbers is again real number.

(ii) Associative Property w.r.t Multiplication: For $a, b, c \in R \Rightarrow a(bc) = (ab)c$

(iii) Multiplicative Identity: For $a \in R$ there is $1 \in R$ such that $a.1 = 1.a = a$ then 1 is called multiplicative identity.

(iv) Multiplicative Inverse: For $a \in R$ where $a \neq 0$ there exists $a^{-1} \in R$ such that $a.a^{-1} = a^{-1}.a = 1$

(v) Commutative Property w.r.t Multiplication: For $a, b \in R \Rightarrow a.b = b.a$

Distributive Property of Multiplication over Addition:

For $a, b, c \in R \Rightarrow a(b + c) = ab + ac$

And $(b + c)a = ba + ca$

Property of Equality of Real Numbers:

(i) Reflexive Property:

For $a \in R \Rightarrow a = a$

(ii) Symmetric Property:

For $a, b \in R$, if $a = b \Rightarrow b + a$

(iii) Transitive Property:

For $a, b, c \in R$ if $a = b$ and $b = c$ then $a = c$

(iv) Additive Property:

For $a, b, c \in R$

If $a = b \Rightarrow a + c = b = c$

(v) Multiplicative Property:

For $a, b, c \in R$ if $a = b \Rightarrow ac = bc$

(vi) Cancellation Property w.r.t Addition:

For $a, b, c \in R$ if $a + c = b + c$ then $a = b$

Properties of Inequality of Real Numbers:

(i) Trichotomy Property:

For $a, b \in R$ exactly one of the following is true. $a = b$ or $a < b$ or $a > b$.

(ii) Transitive Property:

For $a, b, c \in R$ if $a > b$ and $b > c$ then $a > c$.

(iii) Additive Property:

For $a, b, c \in R$

$a < b \Rightarrow a + c < b + c$ and $a > b$

$\Rightarrow a + c > b + c$

(iv) Multiplicative Property:

For $a, b, c \in R$

If $c > 0$ and $a < b \Rightarrow ac < bc$

If $c < 0$ and $a < b \Rightarrow ac > bc$

EXERCISE 2.2

Q1: Name the properties used in the following:

i) $1 + (4 + 3) = (1 + 4) + 3$

ii) $5(a + b) = 5a + 5b$

iii) $a + 0 = 0 + a = a$

iv) $5 \times \frac{1}{5} = \frac{1}{5} \times 5 = 1$

Solution:

i) $1 + (4 + 3) = (1 + 4) + 3$

The property used in this equation is associative law w.r.t addition

ii) $5(a + b) = 5a + 5b$

The property used in this equation is distributive law of multiplication over addition

iii) $a + 0 = 0 + a = a$

The property used in this equation is additive identity property

iv) $5 \times \frac{1}{5} = \frac{1}{5} \times 5 = 1$

The property used in this equation is multiplicative inverse

Q2: Write the missing number.

i) $2 + (\_\_\_\_ + 4) = (2 + 6) + 4$

Ans. 6

ii) $7 + (4 + 2) = 13,\ 50(7 + 4) + 2 = \_\_\_\_$

Ans. 13

iii) $9\times(3\times4)=108$, so $(9\times3)\times4=$ ____

Ans. 108

iv) $5\times(8\times9)=(5\times$ _____ $)\times9$

Ans. 8

Q3: Choose the correct option.

i) $8\times(6\times7)=$

(a) $8\times6-7$ (b) $8-(6-7)$

(c) 8×12 (d) $(8\times6)\times7$

Ans. $\boxed{(d)(8\times6)\times7}$

ii) Which one illustrates the associative law of addition?

(a) $3+(2+4)=(4+4)+1$

(b) $3+(2+4)=(3+2)+4$

(c) $3+(2+4)=(5+2)+2$

(d) $3+(2+4)=(2+6)+1$

Ans. $\boxed{(b)\ 3+(2+4)=(3+2)+4}$

iii) Which one illustrates associative law of multiplication?

(a) $4\times(3\times6)=(6\times6)\times2$

(b) $4\times(3\times6)=(3\times12)\times2$

(c) $4\times(3\times6)=(4\times3)\times6$

(d) $4\times(3\times6)=(3\times8)\times3$

Ans. $\boxed{(c)\ 4\times(3\times6)=(4\times3)\times6}$

Q4: Do this without using distributive property.

i) $39\times63+39\times37$

ii) $81\times450+81\times550$

iii) $50\times161-50\times81$

iv) $827\times60-327\times60$

Solution:

i) Given $39\times63+39\times37$

First find $39\times63=2457\rightarrow$(i)

Second find $39\times37=1443\rightarrow$ (*ii*)

Adding (i) and (ii), we get

$2457+1443=3900$

Hence $39\times63+39\times37=3900$

ii) $81\times450+81\times550$

First find $81\times550=36450\rightarrow$(i)

Second find $81\times450=44550\rightarrow$(ii)

Adding (i) and (ii), we get

$36450+44550=81000$

Hence $81\times450+81\times550=81000$

iii) $50\times161-50\times81$

First find $50\times161=8050\rightarrow$ (i)

Second find $50\times81=4050\rightarrow$(ii)

Now subtracting (ii) from (i)

$8050-4050=4000$

iv) $827\times60-327\times60$

First find $827\times60=49620\rightarrow$(i)

Second find $327\times60=19620\rightarrow$(ii)

Now subtracting (ii) from (i)

$49620-19620=30000$

Hence $827\times60-327\times60=30000$

Radicals and Radicands:

If $\sqrt[n]{a}$ is a real number and *n* is a positive integer greater than 1 then $\sqrt[n]{a}=a^{1/n}$ where *n* is called the index of the radical, the symbol ($\sqrt{\ }$) is called radical symbol, a is called radicand. $\sqrt[n]{a}$ is the radical form of the nth root of a, where $a^{1/n}$ is the exponential form of a. If $\sqrt{a}=a^{1/2}$, if *n* is not given then it will be taken as 2.

Index $\longrightarrow \sqrt[n]{a}$ $\longleftarrow$ Radical $\longleftarrow$ Radicand

If $n=3$, then $\sqrt[3]{a}$ and it is called cube root. For example $\sqrt{3},\sqrt[3]{5},\sqrt{2}+\sqrt{3},\sqrt[5]{a}$, $+\sqrt[3]{b}$ are all radical expressions.

Some important radicals given in the table below:

Square roots	Cube roots	Fourth root
$\sqrt{1}=1$	$\sqrt[3]{1}=1$	$\sqrt[4]{1}=1$
$\sqrt{4}=2$	$\sqrt[3]{8}=2$	$\sqrt[4]{16}=2$
$\sqrt{9}=3$	$\sqrt[3]{27}=3$	$\sqrt[4]{81}=3$
$\sqrt{16}=4$	$\sqrt[3]{64}=4$	$\sqrt[4]{256}=4$
$\sqrt{25}=5$	$\sqrt[3]{125}=5$	$\sqrt[4]{625}=5$

EXAMPLE 5

What is the difference between $x^2 = 16$ and $x = \sqrt{16}$?

Solution:

In the first, we are asked what numbers squared are 16, so x can be 4 or -4 as $4^2 = 16$ and $(-4)^2 = 16$. We write this as $x = \pm 4$.

In the second, $x = 4$, because x is the principal square root of 16, which has always a positive value.

Another form of Radicals:

If $\sqrt[n]{a}$ represents a real number and m/n is a positive rational number $n \geq 2$ then

$a^{m/n} = \sqrt[n]{a^m}$ or $\left(\sqrt[n]{a}\right)^m \longrightarrow (1)$

Where $a^{m/n}$ is the exponential form of the expression and $\sqrt[n]{a^m}$ is the radical form. If $m = n$ the equation (1) becomes

$\sqrt[n]{a^n} = a$

EXAMPLE 6

$\sqrt[3]{64} = \sqrt[3]{(4)^3} = 4$

EXAMPLE 7

$\sqrt[3]{-64} = \sqrt[3]{(-4)^3} = -4$

EXAMPLE 8

Simplify $\sqrt{6x}\sqrt{6y^2}$

Solution:

$\sqrt{6x}\ \sqrt{6y^2}$

$= \sqrt{(6x)(6y^2)} = \sqrt{(36y^2)(x)}$

$= \sqrt{36}\sqrt{y^2} = \sqrt{x}\ 6y\sqrt{x}$

$= \sqrt{6^2 x^2 . x} = 6x\sqrt{x}$

EXAMPLE 9

Simplify $2\sqrt{\dfrac{150xy}{3x}}$

Solution:

$\dfrac{2\sqrt{150xy}}{\sqrt{3x}} = 2 . \sqrt{\dfrac{150xy}{3x}}$

$= 2.\sqrt{50y} = 2.\sqrt{25.2y}$

$= 2.\sqrt{25}.\sqrt{2y} = 2.5\sqrt{2y}$

$= 10\sqrt{2y}$

EXERCISE 2.3

Q1: Write down the index and radicand for each of the following expressions:

i) $\sqrt{\dfrac{11}{y}}$ ii) $\sqrt[3]{\dfrac{13}{3x}}$ iii) $\sqrt[5]{ab^2}$

Solution:

i) $\sqrt{\dfrac{11}{y}}$, Index = 2, Radicand = $\dfrac{11}{y}$

ii) $\sqrt[3]{\dfrac{13}{3x}}$, Index = 3, Radicand = $\dfrac{13}{3x}$

iii) $\sqrt[5]{ab^2}$ Index = 5, Radicand = ab^2

Q2: Transform the following radical into exponential forms. Do not simplify.

i) $\sqrt{36}$ ii) $\sqrt{1000}$ iii) $\sqrt[3]{8}$

iv) $\sqrt[n]{q}$ v) $\sqrt{(5-6a^2)^3}$ vi) $\sqrt[3]{-64}$

Solution:

We convert radical form into exponential form as follow:

i) $\sqrt{36} = (36)^{1/2}$

ii) $\sqrt{1000} = (1000)^{1/2}$

iii) $\sqrt[3]{8} = (8)^{1/3}$

iv) $\sqrt[n]{q} = (q)^{1/n}$

v) $\sqrt{(5-6a^2)^3} = (5-6a^2)^{3/2}$

vi) $\sqrt[3]{-64} = (-64)^{1/3}$

Q3: Transform the following exponential form of an expression into radical form.

i) $-7^{\frac{1}{3}}$ ii) $x^{-\frac{3}{2}}$ iii) $(-8)^{\frac{1}{5}}$

iv) $y^{\frac{3}{4}}$ v) $b^{\frac{4}{5}}$ vi) $(3x)^{\frac{1}{9}}$

Solution:

We convert the exponential form into radical form as:

i) $-7^{1/3} = \sqrt[3]{-7}$

ii) $x^{-3/2} = \sqrt{x^{-3}}$

iii) $(-8)^{1/5} = \sqrt[5]{-8}$

iv) $y^{3/4} = \sqrt[4]{y^3}$

v) $b^{4/5} = \sqrt[5]{b^4}$

vi) $(3x)^{1/9} = \sqrt[9]{3x}$

Q4: Simplify:

i) $\sqrt[3]{125x}$ ii) $\sqrt[3]{\frac{8}{27}}$ iii) $\sqrt{\frac{625x^3y^4}{25xy^2}}$

iv) $\sqrt{(3y-5)^2}$ iv) $6\sqrt{18}$ vi) $\sqrt[3]{54x^3y^3z^2}$

Solution:

i) $\sqrt[3]{125x}$

Given $\sqrt[3]{125x} = (125x)^{1/3}$

$= (5\times5\times5\times x)^{1/3}$

$= (5)^{\cancel{3}\times\frac{1}{\cancel{3}}} x^{\frac{1}{3}}$

$= 5x^{1/3} = 5\sqrt[3]{x}$ Ans.

ii) $\sqrt[3]{\frac{8}{27}}$

Given $\sqrt[3]{\frac{8}{27}} \Rightarrow \left(\frac{8}{27}\right)^{1/3}$

$= \left(\frac{2\times2\times2}{3\times3\times3}\right)^{1/3} = \left(\frac{2^3}{3^3}\right)^{1/3}$

$= \left[\left(\frac{2}{3}\right)^3\right]^{1/3} = \left(\frac{2}{3}\right)^{\cancel{3}\times\frac{1}{\cancel{3}}}$

$= \frac{2}{3}$ Ans.

iii) $\sqrt{\frac{625x^3y^4}{25xy^2}}$

Given $\sqrt{\frac{625x^3y^4}{25xy^2}}$

$= \left[\frac{5\times5\times5\times5x^3y^4}{5\times5\times x\,y^2}\right]^{1/2}$

$= \left[\frac{5^4x^3y^4}{5^2xy^2}\right]^{1/2}$

$= \frac{(5)^{\cancel{4}\times\frac{1}{\cancel{2}}}\times(x^3)^{\frac{1}{2}}(y^{\cancel{4}})^{\frac{1}{\cancel{2}}}}{(5^{\cancel{2}})^{\frac{1}{\cancel{2}}}x^{\frac{1}{2}}(y^{\cancel{2}})^{\frac{1}{\cancel{2}}}}$

$= \frac{5^2\times x^{3/2}.y^2}{5\times x^{1/2}y} = 5^{2-1}\times x^{\frac{3}{2}-\frac{1}{2}}.y^{2-1}$

$= 5^1\times x^{\frac{3-1}{2}}y^1 = 5.x^{\frac{\cancel{2}}{\cancel{2}}}y$

$\Rightarrow$ 5xy Ans.

iv) $\sqrt{(3y-5)^2}$

Given $\sqrt{(3y-5)^2} \Rightarrow \left[(3y-5)^2\right]^{\frac{1}{2}}$

$= (3y-5)^{\cancel{2}\times\frac{1}{\cancel{2}}}$

$\Rightarrow$ $3y-5$

v) $6\sqrt{18}$

Given $\sqrt[6]{18}$

$= 6\times\sqrt{2\times3\times3}$

$= 6\times\sqrt{2}\times(3^{\cancel{2}})^{\frac{1}{\cancel{2}}}$

$= 6\times\sqrt{2}\times3 = 6\times3\sqrt{2}$

$= 18\sqrt{2}$ Ans.

vi) $\sqrt[3]{54x^3y^3z^2}$

$= \left[3\times3\times3\times2x^3y^3z^2\right]^{\frac{1}{3}}$

$= \left[3^3\times2x^3y^3z^2\right]^{\frac{1}{3}}$

$= (3^{\cancel{3}})^{\frac{1}{\cancel{3}}}\times2^{\frac{1}{3}}\times(x^{\cancel{3}})^{\frac{1}{\cancel{3}}}(y^{\cancel{3}})^{\frac{1}{\cancel{3}}}(z^2)^{\frac{1}{3}}$

$=3\times2^{\frac{1}{3}}\times x\times y\times z^{\frac{2}{3}}$

$=3xy\sqrt[3]{2z^2}$ Ans.

Laws of Exponents and Indices:

Base, Exponent and value:

In a^n, a is called the base and n is called the exponent. We read a^n as the "nth power of a" or "a to the nth".

Laws of Exponents:

If $a,b\in R$ and $m,n\in N$ the following laws hold:

(i) Multiplication Law of Indices:

If the base is same then the powers are added. $a^m.a^n=a^{m+n}$

(ii) Division Law of Indices:

To divide two expressions with the same base and different exponents are subtract the exponents $\frac{a^m}{a^n}=a^{m-n}$ where $m>n$.

EXAMPLE 14

$\frac{4^7}{4^2}=4^{7-2}=4^5$

$\frac{5^7}{5^3}=5^{7-3}=5^4$

(iii) Power Law of Indices:

Keep the base same and multiply the exponents $(a^m)^n=a^{mn}$

EXAMPLE 15

$(3^3)^2=3^{3\cdot2}=3^6$

$(4^5)^3=4^{5\times3}=4^{15}$

(iv) More Laws of Indices:

To multiply different bases with the same exponents $(ab)^n=a^nb^n$

(v) To divide two expressions with the same exponents $\left(\frac{a}{b}\right)^n=\frac{a^n}{b^n}$ where $b\neq0$.

EXAMPLE 17

$\left(\frac{3}{2}\right)^5=\frac{3^5}{2^5}$

$\left(\frac{4}{5}\right)^2=\frac{4^2}{5^2}$

(vi) Zero and Negative Indices:

Any non-zero number raised to the power zero equal to one if $a\neq0$ then $a^0=1$.

EXERCISE 2.4

Q1: Write the base, exponential and value of the following:

i) $(2)^{-9}=\frac{1}{1024}$ **ii)** $\left(\frac{a}{b}\right)^p=\frac{a^p}{b^p}$

iii) $(-4)^2=16$

Solution:

i) $2^{-9}=\frac{1}{2^9}=\frac{1}{1024}$

Here base = 2, exponent $=-9$, value $=\frac{1}{1024}$

ii) $\left(\frac{a}{b}\right)^p=\frac{a^p}{b^p}$

Base $=\frac{a}{b}$, exponent = p, value $=\frac{a^p}{b^p}$.

iii) $(-4)^2=16$

Base $=-4$, exponent = 2, value = 16

Q2: If a, b denote the real numbers then simplify:

i) $a^3\times a^5$ **ii)** $\left(\frac{b}{a}\right)^{\frac{3}{2}}\left(\frac{b}{a}\right)^{-\frac{3}{2}}$

iii) $(-a)^4\times(-a)^3$ **iv)** $(-2a^2b^3)^3$

v) $a^3(-2b)^2$ **vi)** $(a^2b)(a^2b)$

vii) $\frac{a^0b^0}{2}$ **viii)** $(-3a^2b^2)^2$

ix) $\left(\frac{a^2}{b^4}\right)^{\frac{3}{2}}$

Solution:

i) $a^3\times a^5=a^{3+5}=a^8$ Ans.

ii) $\left(\frac{b}{a}\right)^{3/2}\left(\frac{b}{a}\right)^{-2/3}$

Given $\left(\frac{b}{a}\right)^{\frac{3}{2}}\left(\frac{b}{a}\right)^{-\frac{2}{3}}$

$=\left(\frac{b}{a}\right)^{\frac{3}{2}-\frac{2}{3}}=\left(\frac{b}{a}\right)^{\frac{9-4}{6}}=\left(\frac{b}{a}\right)^{\frac{5}{6}}$ Ans.

iii) $(-a)^4\times(-a)^3$

$=(-a)^{4+3}=(-a)^7=-a^7$ Ans.

iv) $(-2a^2b^3)^3$

$=(-2)^3(a^2)^3(b^3)^3=-8a^6b^9$ Ans.

v) $a^2(-2b)^2$

$=a^2(-2)^2b^2=4a^2b^2$ Ans.

vi) $(a^2b)(a^2b)$

Given $(a^2b)(a^2b)$

$=a^2.a^2.b.b=a^{2+2}b^{1+1}=a^4b^2$ Ans.

vii) $\frac{a^0b^0}{2}$

Given $\frac{a^0b^0}{2}$

$=\frac{(1)(1)}{2}=\frac{1}{2}$ since $a^0=1, b^0=1$

$=\frac{1}{2}$ Ans.

viii) $(-3a^2b^2)^2$

Given $(-3a^2b^2)^2$

$=(-3)^2(a^2)^2(b^2)^2=9a^4b^4$ Ans.

ix) $\left(\frac{a^2}{b^4}\right)^{3/2}$

G   n $\left(\frac{a^2}{b^4}\right)^{3/2}=\frac{(a^2)^{3/2}}{(b^4)^{3/2}}$

$=\frac{(a^{\cancel{2}})^{\frac{3}{\cancel{2}}}}{(b^4)^{\frac{3}{\cancel{2}}}}$

$=\frac{a^3}{(b^2)^3}=\frac{a^3}{b^6}$ Ans.

Q3: Simplify the following:

i) $\frac{7^6}{7^4}$ ii) $\frac{2^4.5^3}{10^2}$

iii) $\left\{\frac{(a+b)^2.(c+d)^3}{(a+b).(c+d)^2}\right\}^3$

iv) $\left(\sqrt[3]{a}\right)^{\frac{1}{2}}$ v) $\sqrt[5]{x^5}.\sqrt[4]{x^4}$

Solution:

i) $\frac{7^6}{7^4}=7^{6-4}=7^2$

$=7\times7=49$ Ans.

ii) $\frac{2^4.5^3}{10^2}=\frac{2^4\times5^3}{(2\times5)^2}$

$=\frac{2^4\times5^3}{2^2\times5^2}$

$=2^4\times2^{-2}\times5^3\times5^{-2}$

$=2^{4-2}\times5^{3-2}$

$=2^2\times5^1=4\times5=20$ Ans.

iii) $\left\{\frac{(a+b)^2.(c+d)^3}{(a+b).(c+d)^2}\right\}^3$

$=\frac{(a+b)^{2\times3}\times(c+d)^{3\times3}}{(a+b)^3\times(c+d)^{2\times3}}$

$=\frac{(a+b)^6\times(c+d)^9}{(a+b)^3\times(c+d)^6}$

$=(a+b)^6\times(a+b)^{-3}.(c+d)^9\times(c+d)^{-6}$

$=(a+b)^{6-3}.(c+d)^{9-6}$

$=(a+b)^3(c+d)^3$ Ans.

iv) $\left(\sqrt[3]{a}\right)^{1/2}=(a^{1/3})^{1/2}$

$=a^{\frac{1}{3}\cdot\frac{1}{2}}=a^{\frac{1}{6}}$ Ans.

v) $\sqrt[5]{x^5}.\sqrt[4]{x^4}=(x^5)^{\frac{1}{5}}\times(x^4)^{\frac{1}{4}}$

$=x^{\cancel{5}\times\frac{1}{\cancel{5}}}\times x^{\cancel{4}\times\frac{1}{\cancel{4}}}$

$=x.x=x^2$ Ans.

Q4: Simplify the following in such a way that no answers should contain

fractional or negative exponents.

i) $\left(\frac{25}{81}\right)^{\frac{1}{2}}$ ii) $\dfrac{(ab)^{\frac{1}{b}}}{\left(\frac{1}{ab}\right)^{\frac{1}{a}}}$

iii) $\dfrac{2^{p+1}.3^{2p-q}.5^{p+q}.6^{q}}{6^{p}.10^{q+2}.15^{p}}$

iv) $\left(\frac{x^p}{x^q}\right)^{p+q}.\left(\frac{x^q}{x^r}\right)^{q+r}.\left(\frac{x^r}{x^p}\right)^{r+p}$

Solution:

i) Given $\left(\frac{25}{81}\right)^{1/2}=\left(\frac{5\times5}{9\times9}\right)^{1/2}$

$=\frac{(5^2)^{1/2}}{(9^2)^{1/2}}=\frac{5^{\cancel{2}\cdot\frac{1}{\cancel{2}}}}{9^{\cancel{2}\cdot\frac{1}{\cancel{2}}}}=\frac{5}{9}$ Ans.

ii) $\dfrac{(ab)^{1/b}}{\left(\frac{1}{ab}\right)^{1/a}}$

$=(ab)^{\frac{1}{b}}\times(ab)^{\frac{1}{a}}$

$=(ab)^{\frac{1}{b}+\frac{1}{a}}=(ab)^{\frac{a+b}{ab}}$

$=a^{\frac{a+b}{ab}}.b^{\frac{a+b}{ab}}$ Ans.

iii) $\dfrac{2^{p-1}.3^{2p-q}.5^{p+q}.6^{q}}{6^{p}.10^{q-2}.15^{p}}$

$=\dfrac{2^{p-1}.3^{2p-q}.5^{p+q}.(2\times3)^{q}}{(2\times3)^{p}.(2\times5)^{q+2}.(3\times5)^{p}}$

$=\dfrac{2^{p-1}.3^{2p-q}.5^{p+q}.2^{q}.3^{q}}{2^{p}.3^{p}.2^{q+2}.5^{q+2}.3^{p}.5^{p}}$

$=\dfrac{2^{p-1}.2^{q}.3^{2p-q}.3^{q}.5^{p+q}}{2^{p}.2^{q+2}.3^{p}.3^{p}.5^{q+2}.5^{p}}$ $\boxed{\because (ab)^n=a^nb^n}$

$=\dfrac{2^{p+q-1}.3^{2p+q-q}.5^{p+q}}{2^{p+q+2}.3^{2p}.5^{q+2+p}}$ $\boxed{\because a^m.a^n=a^{m+n}}$

$=\dfrac{2^{p+q+1}.3^{2p}.5^{p+q}}{2^{p+q+2}.3^{2p}.5^{p+q+2}}$ $\boxed{\because \frac{a^m}{a^n}=a^{m-n}}$

$=2^{(p-q+1)-(p+q+2)}.3^{2p-(2p)}.5^{p+q-(p+q+2)}$

$=2^{\cancel{p}+\cancel{q}+1-\cancel{p}-\cancel{q}-2}.3^{\cancel{2p}-\cancel{2p}}.5^{\cancel{p}+\cancel{q}-\cancel{p}-\cancel{q}-2}$

$=2^{1-2}.3^{0}.5^{-2}$ As $\boxed{3^0=1}$

$=2^{-1}.(1).5^{-2}$

$=\frac{1}{2}.\frac{1}{5^2}=\frac{1}{2}\times\frac{1}{25}=\boxed{\frac{1}{50}}$ Ans.

iv) $\left(\frac{x^p}{x^q}\right)^{p+q}.\left(\frac{x^q}{x^r}\right)^{q+r}.\left(\frac{x^r}{x^p}\right)^{r+p}$

$=(x^p\times x^{-q})^{p+q}\,(x^q\times x^{-r})^{q+r}$

$(x^r\times x^{-p})^{r+p}$ $\boxed{\because \frac{x^m}{x^n}=x^{m-n}}$

$=(x^{p-q})^{p+q}\times(x^{q-r})^{q+r}$ $\boxed{\because x^m.x^n=x^{m+n}}$

$\times\ (x^{r-p})^{r+p}$ $\because (a-b)(a+b)=a^2-b^2$

$=x^{(p-q)(p+q)}.x^{(q-r)(q+r)}.x^{(r-p)(r+p)}$

$=x^{p^2-q^2}.x^{q^2-r^2}.x^{r^2-p^2}$

$=x^{\cancel{p^2}-\cancel{q^2}+\cancel{q^2}-\cancel{r^2}+\cancel{r^2}-\cancel{p^2}}$

$=x^0=1$ Ans.

Q5: Prove that $\left(\dfrac{4^5.64^3.2^3}{8^5.(128)^2}\right)^{\frac{1}{2}}=2.$

Solution:

Taking L.H.S $\left(\dfrac{4^5.64^3.2^3}{8^5.(128)^2}\right)^{\frac{1}{2}}$

$=\left[\dfrac{4^5\times64\times64\times64\times2^3}{(2\times4)^5\times128\times128}\right]^{\frac{1}{2}}$

$=\left[\dfrac{4^5\times64\times64\times64\times2\times2\times2}{2^5\times4^5\times2\times64\times2\times64}\right]^{\frac{1}{2}}$

$=\left[\dfrac{\cancel{4^5}\times\cancel{64}\times\cancel{64}\times64\times\cancel{2}\times\cancel{2}\times2}{2^5\times\cancel{4^5}\times\cancel{2}\times\cancel{64}\times\cancel{2}\times\cancel{64}}\right]^{\frac{1}{2}}$

$=\left(\frac{2\times64}{2^5}\right)^{\frac{1}{2}}=\left(\frac{2\times2^6}{2^5}\right)^{\frac{1}{2}}=\left(\frac{2^7}{2^5}\right)^{\frac{1}{2}}$

$=\left(2^7\times2^{-5}\right)^{\frac{1}{2}}=\left(2^{7-5}\right)^{\frac{1}{2}}=(2^2)^{\frac{1}{2}}=(2)^{\cancel{2}\times\frac{1}{\cancel{2}}}$

$=2=$ R.H.S

Hence L.H.S = R.H.S

which is proved.

Definition of Complex Number:

The sum of real and imaginary numbers is called *complex number*. It is denoted by $Z = a + ib$ where "a" is called the real part and "b" is called the imaginary part of complex number.

Conjugate of Complex Number:

If $Z = a + ib$ then $\overline{Z} = \overline{a + ib} = a - ib$ is called the conjugate of complex number.

Equality of two Complex Numbers:

Let $Z_1 = a + ib$ and $Z_2 = c + id$ be the two complex numbers then $Z_1 = Z_2$ if and only if $a = c$ and $b = d$.

Operations on Complex Numbers:

1. Addition of Complex Numbers:

If $Z_1 = a + ib$ and $Z_2 = c + id$ then their addition is:

$$Z_1 + Z_2 = a + ib + c + id$$
$$= (a + c) + i(b + d)$$

2. Subtraction of Complex Numbers:

Let $Z_1 = a + ib$ and $Z_2 = c + id$ then their subtraction is:

$$Z_1 - Z_2 = (a + ib) - (c + id)$$
$$= a + ib - c - id$$
$$= (a - c) + i(b - d)$$

3. Multiplication of Complex Numbers:

Let $Z_1 = a + ib$ and $Z_2 = c + id$ then

$$Z_1Z_2 = (a + ib)(c + id)$$
$$= ac + iad + ibc + i^2bd \quad \text{As } i^2 = -1$$
$$= ac + iad + ibc - bd$$
$$= (ac - bd) + i(\text{ad+ bc})$$

4. Division of Complex Numbers:

Let $Z_1 = a + ib$ and $Z_2 = c + id$ then the quotient

$$\frac{Z_1}{Z_2} = \frac{a + ib}{c + id} = \frac{a + ib}{c + id} \times \frac{c - id}{c - id}$$

(By rationalization)

$$= \frac{ac - iad + ibc - i^2bd}{(c)^2 - (id)^2}$$
$$= \frac{ac + ibc - iad + bd}{c^2 - i^2d^2} \quad \because i^2 = -1$$
$$= \frac{ac + bd + ibc - iad}{c^2 + d^2}$$
$$= \frac{ac + bd}{c^2 + d^2} + \frac{i(bc - ad)}{c^2 + d^2}$$

EXAMPLE 22

Let $Z_1 = 2 - i$ & $Z_2 = 3 + i$, then find Z_1Z_2.

Solution:

$$Z_1Z_2 = (2 - i)(3 + i)$$
$$= 6 + 2i - 3i - i^2$$
$$= 6 + (2 - 3)i - (-1) \qquad \text{as } i^2 = -1$$
$$= 6 - i + 1 = 7 - i$$

EXAMPLE 23

Let $Z_1 = 3 + 4i$ and $Z_2 = 3 - 2i$ find the quotient $\frac{Z_1}{Z_2}$.

Solution:

$$\frac{Z_1}{Z_2} = \frac{3 + 4i}{3 - 2i}$$
$$= \frac{(3 + 4i)(3 + 2i)}{(3 - 2i)(3 + 2i)}$$

Multiplying and dividing by $3 + 2i$

$$= \frac{9 + 6i + 12i + 8i^2}{(3)^2 - (2i)^2}$$
$$= \frac{9 + 18i - 8}{9 + 4}$$
$$= \frac{1 + 18i}{13}$$
$$= \frac{1}{13} + \frac{18}{13}i$$

EXERCISE 2.5

Q1: Add the following complex numbers:

i) $8 + 9i, 5 + 2i$ **ii)** $6 + 3i, 3 - 5i$

iii) $2i + 3, 8 - 5\sqrt{-1}$

iv) $\sqrt{3} + \sqrt{2}i, 3\sqrt{3} - 2\sqrt{2}i$

Solution:

i) $8 + 9i, 5 + 2i$

Add these two complex numbers

$=(8+9i)+(5+2i)$

$=(8+5)+(9i+2i)$

$=\boxed{13+11i}$ Ans.

ii) $6+3i, 3-5i$

Add these two complex numbers

$=(6+3i)+(3-5i)$

$=(6+3)+(3i-5i)$

$=\boxed{9-2i}$ Ans.

iii) $2i+3, 8-5\sqrt{-1}$ $\quad \because i=\sqrt{-1}$

Given $3+2i, 8-5i$

Add these two complex numbers

$=(3+2i)+(8-5i)$

$=(3+8)+(2i-5i)$

$=\boxed{11-3i}$ Ans.

iv) $\sqrt{3}+\sqrt{2}i,\ 3\sqrt{3}-2\sqrt{2}i$

Add these two complex numbers

$=\left(\sqrt{3}+\sqrt{2}i\right)+\left(3\sqrt{3}-2\sqrt{2}i\right)$

$=\sqrt{3}+3\sqrt{3}+\sqrt{2}i-2\sqrt{2}i$

$=4\sqrt{3}-\sqrt{2}i$ Ans.

Q2: Subtract:

i) $-2+3i$ from $6-3i$

ii) $9+4i$ from $9-8i$

iii) $1-3i$ from $8-i$

iv) $6-7i$ from $6+7i$

Solution:

i) $-2+3i$ from $6-3i$

$=(6-3i)-(-2+3i)$

$=6-3i+2-3i$

$=(6+2)+(-3i-3i)$ $=\boxed{8-6i}$ Ans.

ii) $9+4i$ from $9-8i$

$(9-8i)-(9+4i)$

$=\cancel{9}-8i-\cancel{9}-4i$ $=\boxed{-12i}$ Ans.

iii) $1-3i$ from $8-i$

$(8-i)-(1-3i)$

$=8-i-1+3i=(8-1)+(-i+3i)$

$=\boxed{7+2i}$ Ans.

iv) $6-7i$ from $6+7i$

$(6+7i)-(6-7i)$

$=6+7i-6+7i$

$=6-6+7i+7i=0+14i$

$=\boxed{14i}$ Ans.

Q3: Multiply the following complex numbers:

i) $1+2i, 3-8i$ ii) $2i, 4-7i$

iii) $5-3i, 2-4i$ iv) $\sqrt{2}+i, 1-\sqrt{2}i$

Solution:

i) $1+2i, 3-8i$

Multiply these two complex numbers

$=(1+2i)(3-8i)$

$=3-8i+6i-16i^2$

$=3-2i-16(-1)$ $\quad \because i^2=-1$

$=3-2i+16=3+16-2i$

$=\boxed{19-2i}$ Ans.

ii) $2i, 4-7i$

Multiply these two complex numbers

$=2i\times(4-7i)$

$=8i-14i^2$ $\quad \because i^2=-1$

$=8i-14(-1)=8i+14$

$=\boxed{14+8i}$ Ans.

iii) $5-3i, 2-4i$

Multiply these two complex numbers

$=(5-3i)(2-4i)$

$=10-20i-6i+12i^2$

$=10-26i+12(-1)$ $\quad \because i^2=-1$

$=10-26i-12=10-12-26i$

$=\boxed{-2-26i}$ Ans.

iv) $\sqrt{2}+i, 1-\sqrt{2}i$

Multiply these two complex numbers

$=\left(\sqrt{2}+i\right)\left(1-\sqrt{2}i\right)$

$=\sqrt{2}-\sqrt{4}i+i-\sqrt{2}i^2$

$=\sqrt{2}-2i+i-\sqrt{2}(-1)$

$=\sqrt{2}+i+\sqrt{2}\Rightarrow\sqrt{2}+\sqrt{2}+i$

$=\boxed{2\sqrt{2}+i}$ Ans.

Q4: Divide the first complex number by the second.

i) $Z_1 = 2 + i,\ Z_2 = 5 - i$

ii) $Z_1 = 3i + 4,\ Z_2 = 1 - i$

Solution:

i) Given $Z_1 = 2 + i,\ Z_2 = 5 - i$

$$\frac{Z_1}{Z_2} = \frac{2+i}{5-i} \quad \text{(Multiply by } \frac{5+i}{5+i}\text{)}$$

$$= \frac{2+i}{5-i} \times \frac{5+i}{5+i}$$

$$= \frac{(2+i)(5+i)}{(5-i)(5+i)} = \frac{10+2i+5i+i^2}{(5)^2-(i)^2}$$

$$= \frac{10+7i-1}{25-(-1)} = \frac{10-1+7i}{25+1} \quad \boxed{\because i^2 = -1}$$

$$= \frac{9+7i}{26} = \frac{9}{26} + \frac{7i}{26} \quad \text{Ans.}$$

ii) $Z_1 = 4 + 3i,\ Z_2 = 1 - i$

$$\frac{Z_1}{Z_2} = \frac{4+3i}{1-i} \quad \text{(Multiply by } \frac{1+i}{1+i}\text{)}$$

$$= \frac{4+3i}{1-i} \times \frac{1+i}{1+i} = \frac{(4+3i)(1+i)}{(1-i)(1+i)}$$

$$= \frac{4+4i+3i+3i^2}{(1)^2-(i)^2}$$

$$= \frac{4+7i+3(-1)}{1-(-1)} = \frac{4+7i-3}{1+1}$$

$$= \frac{1+7i}{2} = \boxed{\frac{1}{2} + \frac{7}{2}i} \quad \text{Ans.}$$

Q5: Perform the indicated operations and reduce to the form $a + bi$.

i) $(4-3i)+(2-3i)$ **ii)** $(5-2i)-(4-7i)$

iii) $2i(4-5i)$ **iv)** $(2-3i)+(4-5i)$

Solution:

i) $(4-3i)+(2-3i)$

$= 4-3i+2-3i$

$\Rightarrow 4+2-3i-3i = \boxed{6-6i}$ Ans.

ii) $(5-2i)-(4-7i)$

$\Rightarrow 5-2i-4+7i$

$= 5-4-2i+7i = \boxed{1+5i}$ Ans.

iii) $2i(4-5i)$

$\Rightarrow 2\times 4i - 2i \times 5i$

$= 8i - 10i^2 = 8i - 10(-1)$

$= 8i + 10 = \boxed{10+8i}$ Ans.

iv) $(2-3i)\div(4-5i)$

$$\Rightarrow \frac{2-3i}{4-5i}$$

(Divide and multiply by $4+5i$)

$$= \frac{2-3i}{4-5i} \times \frac{4+5i}{4+5i} = \frac{(2-3i)(4+5i)}{(4-5i)(4+5i)}$$

$$= \frac{8+10i-12i-15i^2}{(4)^2-(5i)^2} \quad \text{As } i^2 = -1$$

$$= \frac{8-2i+15}{16-25i^2} = \frac{23-2i}{16+25}$$

$$= \frac{23-2i}{41} = \frac{23}{41} - \frac{2i}{41} \quad \text{Ans.}$$

Q6: Find the complex conjugate of the following complex numbers.

$-8-3i,\ -4+9i,\ 7+6i,\ \sqrt{5}-i$

Solution:

Let $Z = 8-3i$ then $\bar{Z} = -8+3i$ is the conjugate of $Z = -8-3i$.

Let $Z_2 = -4+9i$ then $\bar{Z} = -4-9i$ is the conjugate of $Z = -4+9i$.

Let $Z = 7+6i$ then $\bar{Z} = 7-6i$ is the conjugate of $Z = 7+6i$.

Let $Z = \sqrt{5}-i$ then $\bar{Z} = \sqrt{5}+i$ is the conjugate of $Z = \sqrt{5}-i$

Review Exercise 2

Q1: Tell whether the following are true or false?

i) $3^{\frac{1}{3}} = \sqrt{3}$ **ii)** $2^{\frac{2}{3}} = \sqrt[3]{4}$

iii) $\sqrt{49} = 7$ **iv)** $\sqrt[3]{27} = x^3$

i) $3^{\frac{1}{3}} = \sqrt{3}$ (This is false)

ii) $2^{\frac{2}{3}} = \sqrt[3]{4}$ $= 2^{\frac{2}{3}} = (2^2)^{\frac{1}{3}}$

$= (4)^{\frac{1}{3}} = \sqrt[3]{4}$ (This is true)

iii) $\sqrt{49} = 7$ (This is true)

iv) $\sqrt[3]{27} = x^3$ (This is false)

Q2: Select the correct answer.

i) The additive inverse of $\sqrt{5}$ is:

✓**(a)** $-\sqrt{5}$ (b) $\frac{1}{\sqrt{5}}$

(c) $\sqrt{-3}$ (d) -5

ii) $2(3+4) = 2\times3 + 2\times4$. here the property used is:

(a) Commutative

(b) Associative

✓**(c) Distributive**

(d) Closure

iii) $\sqrt{-1}\times\sqrt{-1} =$

(a) 1 (b) i ✓**(c) -1** (d) 0

iv) Which of the following represents numbers greater than -3 but less than 6?

(a) $\{x : -3 > x > 6\}$

(b) $\{x : -3 \le x \le 6\}$

✓**(c)** $\{x : -3 < x < 6\}$

(d) $\{x : -3 \ge x \ge 6\}$

v) If $n = 8$ and $16\times2^m = 4^{n-8}$, then $m = ?$

✓**(a) -4** (b) -2 (c) 0 (d) 8

vi) $(i).(-i) =$

✓(a) 1 (b) -2 (c) $-i$ (d) i

vii) The multiplicative identity of real numbers is:

(a) 0 ✓**(b) 1** (c) -1 (d) R

viii) 0 is:

(a) a positive integer

(b) a negative integer

✓**(c) neither positive nor negative**

(d) not an integer

ix) For $i = \sqrt{-1}$, if $3i(2+5i) = x + 6i$, then $x = ?$

(a) 5 ✓**(b) -15** (c) $5i$ (d) $15i$

x) $\sqrt{0} =$

✓**(a) 0** (b) 1

(c) -1 (d) not defined

xi) $\sqrt{-(-9)^2} = ?$ (Note: $i = \sqrt{-1}$) =

(a) 9 (b) $9+i$

(c) $9-i$ ✓**(d)** $9i$

Q3: Simplify each of the following:

i) $\left(-\frac{2}{3}\right)^3$ **ii)** $(-2)^3.(3)^2$

iii) $-3\sqrt{48}$ **iv)** $\frac{5}{\sqrt[3]{9}}$

Solution:

i) Given $\left(-\frac{2}{3}\right)^3 = \frac{(-2)^3}{(3)^3}$

$= \frac{-2\times-2\times-2}{3\times3\times3} = \frac{-8}{27}$ Ans.

ii) $(-2)^3.(3)^2$

$\Rightarrow (-2)^3\times(3)^2$

$= -2\times-2\times-2\times3\times3$

$= -8\times9 = -72$ Ans.

iii) $-3\sqrt{48}$

$\Rightarrow -3\sqrt{16\times3}$

$= -3\times\sqrt{16}\times\sqrt{3} = -3\times4\sqrt{3}$

$= -12\sqrt{3}$ Ans.

iv) $\frac{5}{\sqrt[3]{9}}$

Given $\frac{5}{\sqrt[3]{9}} = \frac{5}{(9)^{1/3}}$

$= \frac{5}{(3^2)^{1/3}} = \frac{5}{(3)^{2/3}}$

Multiply and divide by $\sqrt[3]{3}$

$= \frac{5\sqrt[3]{3}}{\sqrt[3]{3}\,3^{2/3}} = \frac{5\sqrt[3]{3}}{3^{1/3} \times 3^{2/3}}$

$= \frac{5\sqrt[3]{3}}{3^{\frac{1}{3}+\frac{2}{3}}} = \frac{5\sqrt[3]{3}}{3^{\cancel{3}/\cancel{3}}}$

$= \frac{5\sqrt[3]{3}}{3}$ Ans.

Q4: Multiply $8i$, $-8i$

Solution:

Given $8i, -8i$

Multiply both imaginary numbers

$= 8i \times (-8i)$

$= (8i) \times (-8i) = -64i^2$ $\boxed{\because i^2 = -1}$

$= -64(-1) = 64$ Ans.

Q5: Divide $2 - 5i$ by $1 - 6i$.

Solution:

Given that, divide $2-5i$ by $1-6i$

$= \frac{2-5i}{1-6i}$ (Multiply by $\frac{1+6i}{1+6i}$)

$\Rightarrow \frac{2-5i}{1-6i} \times \frac{1+6i}{1+6i}$

$= \frac{(2-5i)(1+6i)}{(1-6i)(1+6i)}$

$= \frac{2+12i-5i-30i^2}{(1)^2-(6i)^2}$

$= \frac{2+7i-30(-1)}{1-36i^2} = \frac{2+7i+30}{1+36}$

$= \frac{2+30+7i}{37}$ $\boxed{\because i^2 = -1}$

$= \frac{32}{37} + \frac{7i}{37}$ Ans.

Q6: Name the property used

$7 \times \frac{1}{7} = \frac{1}{7} \times 7 = 1.$

Solution:

Given $7 \times \frac{1}{7} = \frac{1}{7} \times 7 = 1$

The property used in this equation is the multiplicative inverse property of real numbers.

THINK:

Q7: Use laws of exponents to simplify:

$\frac{(81)^n . 3^5 + (3)^{4n-1} . (243)}{(9^{2n})(3^3)}$

Solution:

Given $\frac{(81)^n . 3^5 + (3)^{4n-1} . (243)}{(9)^{2n} \times (3)^3}$

$= \frac{(3^4)^n \times 3^5 + (3)^{4n-1} \times 3^5}{(3^2)^{2n} \times (3)^3}$

$= \frac{3^{4n} \times 3^5 + 3^{4n-1} \times 3^5}{3^{6n} \times 3^3}$ $\because (a^m)^n = a^{mn}$

$= \frac{3^{4n+5} + 3^{4n-1+5}}{3^{4n+3}} = \frac{3^{4n+5} + 3^{4n+4}}{3^{4n+3}}$

$= \frac{3^{4n+3}.3^2 + 3^{4n+3}.3^1}{3^{4n+3}}$

$= \frac{\cancel{3^{4n+3}}(3^2+3)}{\cancel{3^{4n+3}}} = 9 + 3 = \boxed{12}$ Ans.

***

Additional MCQs of Unit 2:
Real and Complex Numbers

1. π is.......... number.
   (a) Rational (b) Irrational (c) Natural (d) Real number
   ✓ **Ans. (b) Irrational**

2. $\sqrt{9} =$ ....
   (a) ± 3 (b) -3 (c) 3 (d) 4
   ✓ **Ans. (c) 3**

3. $1\times a = a\times 1 = a$ then the property used is..........
   (a) Inverse (b) Multiplicative identity (c) Additive identity
   ✓ **Ans. (c) Multiplicative identity**

4. For $a > b$ if $c < 0$ then..........
   (a) $ac > bc$ (b) $ac < bc$ (c) $ac = bc$ (d) none
   ✓ **Ans. (a) $ac < bc$**

5. If $c > d$ then which is true..........?
   (a) $c < d$ (b) $\frac{1}{c} > \frac{1}{d}$ (c) $\frac{1}{c} < \frac{1}{d}$ (d) none
   ✓ **Ans. (c) $\frac{1}{c} < \frac{1}{d}$**

6. $\sqrt[3]{-64} =$ ....
   (a) 4 (b) -4 (c) -64 (d) none
   ✓ **Ans. (b) -4**

7. If $Z = -2-3i$ then $\bar{Z} =$ ....
   (a) $2-3i$ (b) $2-3i$ (c) $-2+3i$ (d) none
   ✓ **Ans. (c) $-2+3i$**

8. The value of $\bar{i} =$ ....
   (a) i (b) $-i$ (c) -1 (d) 1
   ✓ **Ans. (b) $-i$**

9. $i^7 =$ ....
   (a) i (b) 1 (c) -1 (d) $-i$
   ✓ **Ans. (d) $-i$**

10. The quotient of two complex numbers is..........
    (a) Complex number (b) Real number
    (c) Imaginary number (d) none
    ✓ **Ans. (a) Complex number**

UNIT 3:
LOGARITHM

Definition: The inverse of exponential function is called *logarithmic function.*

Scientific Notation:

A positive number x is written in scientific notation if it is written as the product of a number "a" where a is real number greater than or equal to 1 but less than 10 and power of 10 is integer n i.e. $x = a \times 10^n$ is called scientific notation.

EXAMPLE

Write 7800 in scientific notation.

Solution:

$7800 = 7.8 \times 10^3$ Ans.

EXAMPLE

Write 0.00729 in scientific notation.

Solution:

In scientific notation 0.00729 is written as,

$0.00729 = 7.29 \times 10^{-3}$

We follow the following three steps to write a number in scientific notation.

STEP-1: We move the decimal point to the left so only one non-zero digit remains to the left. This number is less than 10.

STEP-2: We count the number of places from which the decimal point is moved. This number gives n in the definition.

STEP-3: If decimal point is moved to the left then (n) will be positive and if it is moved to the right then (n) will be negative.

How to write a number x from its scientific notation to the standard form?

i) If the exponent of 10 is positive, we move the decimal to the right.

ii) If the exponent of 10 is negative, we move the decimal point to the left.

EXAMPLE 6

Write the following in standard notation:

i) 4.56×10^7 ii) 8.92×10^{-5}

Solution:

i) $4.56 \times 10^7 = 45600000$

The decimal is moved seven places to the right.

ii) $8.92 \times 10^{-5} = 0.0000892$

The decimal is moved five places to the left.

EXERCISE 3.1

Q1: Write each number in scientific notation:

i) 405,000 ii) 1,670,000

iii) 0.00000039 iv) 0.00092

v) 234,600,000,000

vi) 8,904,000,000

vii) 0.00104 viii) 0.00000000514

ix) 0.05×10^{-3}

Solution:

i) $405,000 \Rightarrow 4.05 \times 10^5$

Which is in scientific notation

ii) $1,670,000 \Rightarrow 1.67 \times 10^6$

Which is in scientific notation

iii) $0.00000039 \Rightarrow 3.9 \times 10^{-7}$

Which is in scientific notation

iv) $0.00092 \Rightarrow 9.2 \times 10^{-4}$

Which is in scientific notation

v) $234,600,000,000 \Rightarrow 2.34 \times 10^{11}$

Which is in scientific notation

vi) $8,904,000,000 \Rightarrow 8.9 \times 10^9$

Which is in scientific notation

vii) $0.00104 \Rightarrow 1.04 \times 10^{-3}$

Which is in scientific notation

viii) $0.00000000514 \Rightarrow 5.14 \times 10^{-9}$

Which is in scientific notation

ix) $0.05 \times 10^{-3} \Rightarrow 5 \times 10^{-2} \times 10^{-3}$

$\Rightarrow 5 \times 10^{-5}$

Which is in scientific notation

Q2: Write each number in standard notation:

i) 8.3×10^{-5} ii) 4.1×10^6

iii) 2.07×10^7 iv) 3.15×10^{-6}

v) 6.27×10^{-10} vi) 5.41×10^{-8}

vii) 7.632×10^{-4} **viii)** 9.4×10^{5}

ix) -2.6×10^{9}

Solution:

i) Given 8.3×10^{-5}

$=\frac{83}{10}\times\frac{1}{10^5}=\frac{83}{10}\times\frac{1}{100000}$

$=\frac{83}{1000000}=\boxed{0.000083}$ Ans.

ii) Given 4.1×10^{6}

$=\frac{41}{1\cancel{0}}\times100000\cancel{0}$

$=\boxed{41000000}$ Ans.

iii) Given 2.07×10^{7}

$=\frac{207}{1\cancel{00}}\times100000\cancel{00}$

$=\boxed{20700000}$ Ans.

iv) Given 3.15×10^{-6}

$=\frac{315}{100}\times\frac{1}{10^6}=\frac{315}{100}\times\frac{1}{1000000}$

$=\frac{315}{100000000}$

$=\boxed{0.00000315}$ Ans.

v) Given 6.27×10^{-10}

$=\frac{627}{100}\times\frac{1}{10^{10}}$

$=\frac{627}{100}\times\frac{1}{10000000000}$

$=\frac{627}{1000000000000}$

$=\boxed{0.000000000627}$ Ans.

vi) Given 5.41×10^{-8}

$=\frac{541}{100}\times\frac{1}{10^8}$

$=\frac{541}{100\times100000000}$

$=\frac{541}{10000000000}$

$=\boxed{0.0000000541}$ Ans.

vii) Given 7.632×10^{-4}

$=\frac{7632}{1000}\times\frac{1}{10^4}$

$\Rightarrow \frac{7632}{1000}\times\frac{1}{10000}$

$=\frac{7632}{10000000}=\boxed{0.0007632}$ Ans.

viii) Given 9.4×10^{5}

$=\frac{94}{\cancel{10}}\times10000\cancel{0}$

$=\boxed{940000}$ Ans.

ix) Given -2.6×10^{9}

$=-\frac{26}{\cancel{10}}\times100000000\cancel{0}$

$=\boxed{-2600000000}$ Ans.

Q3: How long does it take to travel to earth from the sun? The sun is 9.3×10^7 *mi* from earth and light travel 1.8×10^5 *mi/s*.

Solution:

Given: Distance of sun from earth is

$S=9.3\times10^7\,mi$

Speed of light is

$v=1.86\times10^5\,mi/s$

Time required $=t=?$

We know from speed formula

$S=Vt \Rightarrow t=\frac{S}{V}\longrightarrow(1)$

$\Rightarrow t=\frac{9.3\times10^7}{1.86\times10^5}$

$=5\times10^{7-5}=5\times10^2$

$t=5\times100=500$ second

Now $\frac{500}{60}=\frac{50}{6}=8.30=8$ 8 min and 30sec

$\therefore$ Required time is $\boxed{t=8\,\text{min } and \text{ } 30\,\text{sec}}$

Definition:

If for a positive real number a (but $a\neq1$) $a^x=y$ then the index x is called the logarithm of y to the base "a". It is written as

$\log_a y=x$

We can write,

$\log_a y=x$ if any only if $a^x=y$

Here $a^x=y$ is called the exponential form and $\log_a y$ is the logarithmic form.

EXAMPLE 8

Write the following in logarithmic form:

i) $2^4 = 16$ ii) $4^{-3} = \frac{1}{64}$

Solution:

We know that $a^x = y \Rightarrow \log_a y = x$ ....(*)

i) $2^4 = 16$

$\because a = 2,\ x = 4,\ y = 16$

So using the above result (*)

$2^4 = 16$ is equivalent to $\log_2 16 = 4$

ii) $4^{-3} = \frac{1}{64}$

here $a = 4,\ x = -3,\ y = \frac{1}{64}$, so using the result (*)

$4^{-3} = \frac{1}{64}$ is equivalent to $\log_4\left(\frac{1}{64}\right) = -3$

EXAMPLE 9

Write the following in exponential form

i) $\log_8(64) = 2$ ii) $\log_3\left(\frac{1}{9}\right) = -2$

Solution:

We know that $a^x = y \Rightarrow \log_a y = x$ ....(*)

i) $\log_8(64) = 2$

Here $a = 8,\ y = 64$ and $x = 2$, so using the result (*)

$\therefore \log_8(64) = 2$ is equal to $8^2 = 64$.

ii) $\log_3\left(\frac{1}{9}\right) = -2$

Here $a = 3,\ y = \frac{1}{9}$ and $x = -2$, so using the result (*)

$\therefore \log_3\left(\frac{1}{9}\right) = -2$ is equal to $3^{-2} = \frac{1}{9}$.

EXERCISE 3.2

Q1: Write the following in logarithmic form:

i) $4^4 = 256$ ii) $2^{-6} = \frac{1}{64}$

iii) $10^0 = 1$ iv) $x^{3/4} = y$

v) $64^{2/3} = 16$ vi) $64^{2/3} = 16$

Solution:

i) $4^4 = 256$

Formula is, $a^x = y \Rightarrow \log_a y = x$

$\Rightarrow \log_4 256 = 4$ Ans.

ii) $2^{-6} = \frac{1}{64}$

Formula is, $a^x = y \Rightarrow \log_a y = x$

$\Rightarrow \log_2 \frac{1}{64} = -6$ Ans.

iii) $10^0 = 1$

Formula is, $a^x = y \Rightarrow \log_a y = x$

$\Rightarrow \log_{10} 1 = 0$ Ans.

iv) $x^{3/4} = y$

Formula is, $a^x = y \Rightarrow \log_a y = x$

$\Rightarrow \log_x y = \frac{3}{4}$ Ans.

v) $3^{-4} = \frac{1}{81}$

Formula is, $a^x = y \Rightarrow \log_a y = x$

$\Rightarrow \log_3 \frac{1}{81} = -4$ Ans.

vi) $64^{2/3} = 16$

Formula is, $a^x = y \Rightarrow \log_a y = x$

$\Rightarrow \log_{64} 16 = \frac{2}{3}$ Ans.

Q2: Write the following in exponential form:

i) $\log_a\left(\frac{1}{a^2}\right) = -2$ ii) $\log_2 \frac{1}{128} = -7$

iii) $\log_b 3 = 64$ iv) $\log_a a = 1$

v) $\log_a 1 = 0$ vi) $\log_4 \frac{1}{8} = \frac{-3}{2}$

Solution:

i) $\log_a\left(\frac{1}{a^2}\right) = -2$

Formula is, $\log_a y = x \Rightarrow a^x = y$

$\Rightarrow \quad a^{-2} = \frac{1}{a^2}$ Ans.

ii) $\log_2 \frac{1}{128} = -7$

$\Rightarrow \quad 2^{-7} = \frac{1}{128}$ Ans.

iii) $\log_b 3 = 64$

$\Rightarrow \quad b^{64} = 3$ Ans.

iv) $\log_a a = 1$

$\Rightarrow \quad a^1 = a$ Ans.

v) $\log_a 1 = 0$

$\Rightarrow \quad a^0 = 1$ Ans.

vi) $\log_4 \frac{1}{8} = \frac{-3}{2}$

$\Rightarrow \quad 4^{\frac{-3}{2}} = \frac{1}{8}$ Ans.

Q3: Solve:

i) $\log_{\sqrt{5}} 125 = x$ ii) $\log_4 x = -3$

iii) $\log_{81} 9 = x$ iv) $\log_3(5x+1) = 2$

v) $\log_2 x = 7$ vi) $\log_x 0.25 = 2$

vii) $\log_x(0.001) = -3$ viii) $\log_x \frac{1}{64} = -2$

ix) $\log_{\sqrt{3}} x = 16$

Solution:

i) $\log_{\sqrt{5}} 125 = x$

As $\log_a y = x \Rightarrow a^x = y$

$\Rightarrow (\sqrt{5})\text{x} = 125$

$\Rightarrow (5^{\frac{1}{2}})\text{x} = 5^3$

$\Rightarrow 5^{\frac{x}{2}} = 5^3$

Since the bases are same so the powers must be equal.

$\Rightarrow \frac{x}{2} = 3$

$\Rightarrow \boxed{x = 6}$ Ans.

ii) $\log_4 x = -3$

$\because \log_a y = x \Rightarrow a^x = y$

$\Rightarrow (4)^{-3} = x$

$\Rightarrow x = \frac{1}{(4)^3} = \frac{1}{64}$

$\Rightarrow \boxed{x = 1/64}$ Ans.

iii) $\log_{81} 9 = x$

$\because \log_a y = x \Rightarrow a^x = y$

$\Rightarrow (81)^x = 9 \Rightarrow (3^4)^x = 3^2$

$3^{4x} = 3^2$

$\Rightarrow 4x = 2$

$\Rightarrow x = \frac{2}{4} = \frac{1}{2}$

$\Rightarrow \boxed{x = 1/2}$ Ans.

iv) $\log_3(5x+1) = 2$

$\because \log_a y = x \Rightarrow a^x = y$

$\Rightarrow 3^2 = 5x+1 \Rightarrow 9 = 5x+1$

$5x+1 = 9 \Rightarrow 5x = 9-1 = 8$

$\Rightarrow \boxed{x = 8/5}$ Ans.

v) $\log_2 x = 7$

$\because \log_a y = x \Rightarrow a^x = y$

$\Rightarrow 2^7 = x$

$\Rightarrow 128 = x$

$\Rightarrow \boxed{x = 128}$ Ans.

vi) $\log_x 0.25 = 2$

$\because \log_a y = x \Rightarrow a^x = y$

$\Rightarrow x^2 = 0.25$

$\Rightarrow x^2 = \frac{25}{100} = \left(\frac{5}{10}\right)^2$

$\Rightarrow \sqrt{x^2} = \sqrt{\left(\frac{5}{10}\right)^2}$

Taking square root

$\Rightarrow x = \frac{\cancel{5}}{\cancel{10}_2} \Rightarrow \boxed{x = \frac{1}{2}}$ Ans.

vii) $\log_x(0.001) = -3$

$\because \log_a y = x \Rightarrow a^x = y$

$\Rightarrow x^{-3} = 0.001$

$\Rightarrow x^{-3} = \dfrac{1}{1000}$

$\Rightarrow x^{-3} = \left(\dfrac{1}{10}\right)^3$

$\Rightarrow x^{-3} = 10^{-3}$

$\Rightarrow \boxed{x = 10}$ Ans.

viii) $\log_x \dfrac{1}{64} = -2$

$\because \log_a y = x \Rightarrow a^x = y$

$\Rightarrow x^{-2} = \dfrac{1}{64}$

$\Rightarrow x^{-2} = \left(\dfrac{1}{8}\right)^2$

$\Rightarrow x^{-2} = \left(\dfrac{1}{2^3}\right)^2 = \dfrac{1}{(8)^2}$

$\Rightarrow x^{-2} = (8)^{-2} \Rightarrow \boxed{x = 8}$ Ans.

ix) $\log_{\sqrt{3}} x = 16$

$\because \log_a y = x \Rightarrow a^x = y$

$\Rightarrow \left(\sqrt{3}\right)^{16} = x$

$\Rightarrow x = (3^{\frac{1}{2}})^{16}$

$= (3^{\frac{1}{\cancel{2}}})^{\cancel{16}\,8} \Rightarrow x = 3^8 = 6560$

$\Rightarrow \boxed{x = 6560}$ Ans.

a) Common Logarithm:

That logarithm which has base 10 is known as common or Briggs logarithm. This was introduced by a British mathematician Professor Henry Briggs.

Note that;

$\log_{10} x = \log x$ like $\log_2 100 = \log 100$

b) Characteristic and Mantissa:

Logarithm of a number consists of two parts. The digit before the decimal point is called the characteristic of log and the decimal fraction part is called the mantissa of the log. Mantissa is always positive.

For example,

$10^{1.5377} = 34.49$

Hence $\log 34.49 = 1.5377$

Here characteristic is 1 and mantissa is 0.5377.

Rules to find Characteristic of logx:

1. If $x > 1$, characteristic = (No of digits on left side of decimal point in x) − 1
2. If $x < 1$, characteristic = − (No of zeros immediately following the decimal point) + 1.

The use of logarithmic table to find the log of a number:

To find the logarithm of a number we have to find the characteristic and the mantissa. The characteristic is found by the above two rules. The mantissa is found from the logarithmic table.

EXAMPLE

Write the characteristics of the following logarithms:

i) log 4350	**ii) log 435**
iii) log 43.5	**iv) log 4.35**
v) log 0.435	**vi) log 0.0435**
vii) log 0.00435	**viii) log 0.000435**

Solution:

i) Characteristic of $\log 4350 = 3$ $(\because 4 - 1 = 3)$

ii) Characteristic of $\log 435 = 2$ $(\because 3 - 1 = 2)$

iii) Characteristic of $\log 43.5 = 1$ $(\because 2 - 1 = 1)$

iv) Characteristic of $\log 4.35 = 0$ $(\because 1 - 1 = 0)$

v) Characteristic of $\log 0.435 = -1$ $(\because -(0 + 1) = -1)$

vi) Characteristic of $\log 0.0435 = -2$ $(\because -(1 + 1) = -2)$

vii) Characteristic of $\log 0.00435 = -3$ $(\because -(2 + 1) = -3)$

viii) Characteristic of $\log 0.000435 = -4$ $(\because -(3 + 1) = -4)$

EXERCISE 3.3

Q1: Find the characteristic of the common logarithm of each of the following numbers:

i) 57 ii) 7.4 iii) 5.63 iv) 982.5 v) 7824 vi) 186000 vii) 0.71

Solution:

i) 57

Given $57 = 5.7\times10^1$

Characteristic $=1$

ii) 7.4

Given $7.4 = 7.4\times10^0$

Characteristic $=0$

iii) 5.63

Given $5.63 = 5.63\times10^0$

Characteristic $=0$

iv) 982.5

Given $982.5 = 9.825\times10^2$

Characteristic $=2$

v) 7824

Given $7824 = 7.824\times10^3$

Characteristic $=3$

vi) 186000

Given $186000 = 1.86\times10^5$

Characteristic $=5$

vii) 0.71

Given $0.71 = 7.1\times10^{-1}$

Characteristic $=-1$

Q2: Find the following:

i) log87.2 ii) log37300 iii) log753 iv) log9.21 v) log0.00159 vi) log0.0256 vii) log6.753

Solution:

i) log87.2

As $\log 87.2 \Rightarrow 8.72\times10^1$

The characteristic is $=1$

For mantissa see row 87 and column 2 of the log table.

The mantissa of $\log 87.2 = 0.9405$

Add characteristic 1 with 0.9405

Hence $\log 87.2 = 1.9405$ or $\boxed{1.941}$ Ans.

ii) log37300

As $\log 37300 \Rightarrow 3.73\times10^4$

The characteristic is $=4$

For mantissa see row 37 and column 3 of log table.

The mantissa of $\log 37300 = 0.5717$

Add 4 with mantissa, we get

Hence $\log 37300 = 4.5717$ or $\boxed{4.572}$ Ans.

iii) log753

As $\log 753 \Rightarrow 7.53\times10^2$

The characteristic is $=2$

For mantissa see row 75 and column 3 of log table.

The mantissa of $\log 753 = 0.8768$

Hence $\log 753 = \boxed{2.8768}$ or $\boxed{2.877}$ Ans.

iv) log9.21

As $9.21 = 9.21 \times 10^0$

The characteristic is $=0$

For mantissa see row 92 in column 1 in log table.

The mantissa of $\log 9.21 = 0.9642$

Hence $\log 9.21 = \boxed{0.9642}$ Ans.

v) log0.00159

As $0.00159 = 1.59\times10^{-3}$

The characteristic is $=-3$ or $\bar{3}$

For mantissa see row 15 in column 9 in log table.

The mantissa of $\log 0.00159 = 0.2014$

Hence $\log 0.00159 = \boxed{\bar{3}.2014}$ Ans.

vi) log0.0256

As $0.0256 = 2.56\times10^{-2}$

The characteristic is $=-2$ or $\bar{2}$

For mantissa see row 25 and column 6 and the mantissa $=0.4082$

Hence $\log 0.0256 = \boxed{\bar{2}.4082}$ Ans.

vii) log6.753

As $6.753 \Rightarrow 6.753\times10^0$

The characteristic is $=0$

For mantissa see row 67 and column 5 which is 0.8293 and then difference column 3 which is 2.

Add 2 with 0.8293

Which is $0.8293 + 2 = 0.8295$
Hence log $6.753 = 0.8295$ Ans.

Q3: Find logarithms of the following numbers:

i) 2476 ii) 2.4 iii) 92.5
iv) 482.7 v) 0.783 vi) 0.09566
vii) 0.006735 viii) 700

Solution:

i) 2476

As $2476 = 2.476 \times 10^3$
The characteristic = 3
We search 24 in 7 column of the log table which is 3927. Now search in difference column 7 which is 11. Add 11 to 3927
$\Rightarrow 3927 + 11 = 3938$
Thus mantissa = 0.3938
Hence log $2476 = \boxed{3.3938}$ Ans.

ii) 2.4

As $2.4 = 2.4 \times 10^0$
The characteristic = 0
We find mantissa
See 24 in 0 column of log table, so
Mantissa = 0.3802
The mantissa = 0.3802
Hence $\log 2.4 = \boxed{0.3802}$ Ans.

iii) 92.5

As $92.5 = 9.25 \times 10^1$
Characteristic = 1
We find mantissa:
See 92 in column 5 of log table, the value is 9661
So mantissa = 0.9661
Hence $\log 92.5 = \boxed{1.9661}$ Ans.

iv) 482.7

As $482.7 = 4.827 \times 10^2$
Characteristic = 2
We find mantissa:
See 48 in column 2 of log table, the value is 6830 and see the difference column 7 which is 6.
Now add 6 with 6830
So $6830 + 6 = 6836$
Thus mantissa = 0.6836
Hence $\log 482.7 = \boxed{2.6836}$ Ans.

v) 0.783

As $0.783 = 7.83 \times 10^{-1}$
Characteristic $= -1$ or $\bar{1}$
We find mantissa:
We see 78 in column 3 in log table which is 8938
$\therefore$ Mantissa = 0.8938
Hence $\log 0.783 = \boxed{\bar{1}.8938}$ Ans.

vi) 0.09566

As $0.09566 = 9.566 \times 10^{-2}$
Characteristic $= \bar{2}$
We search 95 in column 6 of log table which is 9805. Now search in difference column 6 which is 3. Add 3 with 9805 which is $9805 + 3 = 9808$
$\therefore$ Mantissa = 0.9808
Hence $\log 0.09566 = \boxed{\bar{2}.9808}$ Ans.

vii) 0.006735

As $0.006735 = 6.735 \times 10^{-3}$
Characteristic $= -3$ or $\bar{3}$
See 67 in column 3 of log table, the value is 8280. Now see in difference column 5 the value is 3. Add 3 to 8280 which is $8280 + 3 = 8283$
$\therefore$ Mantissa = 0.8283
Hence $\log 0.8283 = \boxed{\bar{3}.8283}$ Ans.

viii) 700

As $700 = 7.00 \times 10^2$
Characteristic = 2
We see 70 in column 0 of log table which is 8451.
$\therefore$ Mantissa = 0.8451
Hence $\log 700 = \boxed{2.8451}$ Ans.

Anti-Logarithm:

Definition: If $y = \log_{10} x$, then x is called the anti-logarithm of y to the base 10 and it is written as $x = \text{antilog}y$. To find the value of y we use the table of anti-logarithm. i.e.
$y = \text{antilog}x$

EXAMPLE (13)

Find the numbers whose logarithms are

(i) 2.3456 (ii) $\bar{2}.1576$

Solution:

i) 2.3456

To find the sequence of digits in the required number, we look up the table of antilogarithms given at the end of the book.

Since the mantissa is .3456, we look out for the row in the first column containing the first two digits of the mantissa .34. and in this row we select the number 2213 in the column headed by the third digit 5 of the mantissa. Then, add to this the figure 3 which is the fourth digit value from 6^{th} column of mean differences in the same row.

The required sequence of digits in the number is $(2213+3=2216)$. As the characteristic of the given logarithm is 2, there must be $2+1$ i.e. 3 digits in the integral part of the number.

Hence the number whose logarithm is 2.3456 must be 221.6.

ii) $\bar{2}.1576$

As in above, from the table of antilogarithms, the sequence of digits in the number, corresponding to the mantissa .1576, is found to be 1437. Now the characteristic is -2. Therefore, there must be one zero between the decimal point and the first significant digit in the number.

Hence the number whose logarithm is $\bar{2}.1576$ must be .01437.

EXERCISE 3.4

Q1: Find anti-logarithm of the following numbers:

i) 1.2508	**ii) 0.8401**
iii) 2.540	**iv) $\bar{2}.2508$**
v) $\bar{1}.5463$	**vi) 3.5526**

Solution:

i) 1.2508

Here characteristic = 1

And mantissa = 0.2508

To find anti-logarithm we use anti-logarithm table. We see 0.25 in column 0 in anti-logarithm table which is 1778.

And now see in difference column 8 which is 3. Now add 3 to 1778 which is 1781.

Since characteristic is 1 then place decimal after two digits.

Hence anti-log(1.2508) = $\boxed{17.81}$ Ans.

ii) 0.8401

Here characteristic = 0

And mantissa = 0.8401

To find anti-logarithm we use anti-logarithm table. We see 0.84 in column 0 in anti-logarithm table which is 6918. And now see in difference column 1 which is 2. Now add 2 to 6918 which is 6920.

Since characteristic is 0 then place decimal after one digits.

Hence anti-log (0.8401) = $\boxed{6.920}$ Ans.

iii) 2.540

Here characteristic = 2

And mantissa = 0.540

To find anti-logarithm we use anti-logarithm table. We see 0.54 in column 0 in anti-logarithm table which is 3467.

Since characteristic is 2 then place decimal after three digits.

Hence anti-log(2.540) = $\boxed{346.7}$ Ans.

iv) $\bar{2}.2508$

Here characteristic = $\bar{2}$

And mantissa = 0.2508

To find anti-logarithm we use anti-logarithm table. We see 0.25 in column 0

in anti-logarithm table which is 1778. And now see in difference column 8 which is 3. Now add 3 to 1778 which is 1781.
Since characteristic is $\bar{2}$ then place decimal point before one zero.

Hence anti-log $(\bar{2}.2508) = \boxed{0.01781}$ Ans.

v) $\bar{1}.5463$

Here characteristic $= \bar{1}$
And mantissa $= 0.5463$
To find anti-logarithm we use anti-logarithm table. We see 0.54 in column 6 in anti-logarithm table which is 3516. And now see in difference column 3 which is 2. Now add 2 to 3516 which is 3518.
Since characteristic is $\bar{1}$ then place decimal before digits.

Hence anti-log $(\bar{1}.5463) = \boxed{0.3518}$ Ans.

vi) 3.5526

Here characteristic $= 3$
And mantissa $= 0.5526$
To find anti-logarithm we use anti-logarithm table. We see 0.55 in column 2 in anti-logarithm table which is 3565. And now see in difference column 6 which is 5. Now add 5 to 3565 which is 3570.
Since characteristic is 3 then place decimal after four digits.

Hence anti-log $(3.5526) = \boxed{3570.0}$ Ans.

Q2: Find the values of x from the following equations:

i) $\log x = \bar{1}.8401$ **ii)** $\log x = 2.1931$

iii) $\log x = 4.5911$ **iv)** $\log x = \bar{3}.0253$

v) $\log x = 1.8716$ **vi)** $\log x = \bar{2}.8370$

Solution:

i) $\log x = \bar{1}.8401$

Here characteristic $= \bar{1}$
And mantissa $= 0.8401$
To find anti-logarithm we use anti-logarithm table. We see 0.84 in column 0 in anti-logarithm table which is 6918. And now see in difference column 1 which is 2. Now add 2 to 6918 which is 6920.
Since characteristic is $\bar{1}$ then place decimal before digits.

Hence anti-log$(\log x) = $ anti-log$(\bar{1}.8401)$

$\Rightarrow \boxed{x = 0.6920}$ Ans.

ii) $\log x = 2.1931$

Here characteristic $= 2$
And mantissa $= 0.1931$
To find anti-logarithm we use anti-logarithm table. We see 0.19 in column 3 in anti-logarithm table which is 1560. And now see in difference column 1 which is 0. Now add 0 to 1560 which is 1560.
Since characteristic is 2 then place decimal after three digits.
Hence anti-log$(\log x) = $ anti-log(2.1931)

$\Rightarrow \boxed{x = 156.0}$ Ans.

iii) $\log x = 4.5911$

Here characteristic $= 4$
And mantissa $= 0.5911$
To find anti-logarithm we use anti-logarithm table. We see 0.59 in column 1 in anti-logarithm table which is 3899. And now see in difference column 1 which is 1. Now add 1 to 3899 which is 3900.
Since characteristic is 4 then place decimal after four digits.
Hence anti-log$(\log x) = $ anti-log(4.5911)

$\Rightarrow \boxed{x = 39000}$ Ans.

iv) $\log x = \bar{3}.0253$

Here characteristic $= \bar{3}$
And mantissa $= 0.0253$
To find anti-logarithm we use anti-logarithm table. We see 0.02 in column 5 in anti-logarithm table which is 1059. And now see in difference column 3 which is 1. Now add 1 to 1059 which is 1060.
Since characteristic is $\bar{3}$ then place decimal point before two zeros.
Hence anti-log$(\log x) = $ anti-log $(\bar{3}.0253)$

$\Rightarrow \boxed{x = 0.001060}$ Ans.

v) $\log x = 1.8716$

Here characteristic $= 1$
And mantissa $= 0.8716$
To find anti-logarithm we use anti-

logarithm table. We see 0.87 in column 1 in anti-logarithm table which is 7430. And now see in difference column 6 which is 10. Now add 10 to 7430 which is 7440.

Since characteristic is 1 then place decimal after two digits.

Hence anti-log(log$x$) = anti-log(1.8716)

$\Rightarrow \boxed{x = 74.40}$ Ans.

vi) $\log x = \bar{2}.8370$

Here characteristic $= \bar{2}$

And mantissa $= 0.8370$

To find anti-logarithm we use anti-logarithm table. We see 0.83 in column 7 in anti-logarithm table which is 6871.

Since characteristic is $\bar{2}$ then place decimal point before one zero.

Thus anti-log(log$x$) = anti-log ($\bar{2}$.8370)

$\Rightarrow \boxed{x = 0.6871}$ Ans.

Common and Natural Logarithm:

There are two system of logarithms used in mathematics.

i) System of Common Logarithm:

When the base of logarithm $a = 10$ then such system is called system of common logarithm. It was invented by Henry Briggs. When there is no base of the logarithm then it must be understood that it has base 10. Thus common log can also be written as,

$\log_{10} x = \log x$ for example $\log(204)$ means that $\log_{10}(204)$

ii) System of Natural Logarithm:

When the base of the logarithm $a =$ "e" then such a system is called system of natural log. This log is written as $\log_e x = \ell n x$

Here constant "e" is called Euler constant and its value is $\boxed{e = 2.71828}$

Laws of Logarithm:

There are four laws of logarithms. We shall prove them one by one.

1. $\log_a mn = \log_a m + \log_a n$

Proof: Let $\log_a m = x \Rightarrow a^x = m \rightarrow (1)$

(By definition)

Let $\log_a m = y \Rightarrow a^y = n \rightarrow (2)$

Multiply (1) and (2)

$mn = a^x . a^y = a^{x+y}$

$\Rightarrow a^{x+y} = mn = x + y$

From (1) and (2)

$\log_a mn = \log_a m + \log_a n$

2. $\log_a \frac{m}{n} = \log_a m - \log_a n$

Proof: As $\frac{m}{n} = \frac{a^x}{a^y} = a^{x-y}$

$\Rightarrow a^{x-y} = \frac{m}{n}$

From (1) and (2)

$\Rightarrow \log_a \frac{m}{n} = x - y$

$\Rightarrow \log_a \frac{m}{n} = \log_a m - \log_a n$

3. $\log_a m^n = n \log_a m$

Proof: From equation (1) $a^x = m$,

Taking power n on both sides

$(a^x)^n = (m)^n \Rightarrow a^{nx} = m^n$

$\Rightarrow \log_a m^n = nx$

Use equation (1) we get

$\log_a m^n = n \log_a m$

4. $\log_a m \log_a n = \log_a n$

EXAMPLE 14

Express each of the following as a single logarithm.

i) $\log 2 + \log 3$ **ii)** $\log 6 - \log 2$

iii) $-1 + \log y$ **iv)** $\log_2 3 . \log_3 5$

v) $\log_2 8 + \log_2 32$

vi) $\log 3 + \log 5 + \log 6 - \log 25$

Solution:

i) $\log 2 + \log 3 = \log(2 \times 3) = \log 6$

ii) $\log 6 - \log 2 = \log \frac{6}{3} = \log 2$

iii) $-1 + \log y$

$= -\log 10 + \log y = \log 10^{-1} + \log y$

$= \log\frac{1}{10} + \log y = \log 0.10 + \log y$

$= \log 0.10y$

iv) $\log_2 3 . \log_3 5 = \log_2 5$

v) $\log_2 8 + \log_2 32$

$= \log_2(8\times 32) = \log_2 256$

vi) $\log 3 + \log 5 + \log 6 - \log 25$

$= \log\frac{3\times 5\times 6}{25} = \log\frac{90}{25} = \log\frac{18}{5}$

EXERCISE 3.5

Q1: Use logarithm properties to simplify the expression:

i) $\log_7 \sqrt{7}$ ii) $\log_8 \frac{1}{2}$

iii) $\log_{10}\sqrt{1000}$ iv) $\log_9 3 + \log_9 27$

v) $\log\frac{1}{(0.0035)^{-4}}$ vi) $\log 45$

Solution:

i) $\log_7\sqrt{7}$

$\log_7\sqrt{7} = x$

$\Rightarrow 7^x = \sqrt{7}$

$\Rightarrow 7^x = (7)^{\frac{1}{2}}$

Comparing power on both sides

$\Rightarrow x = \frac{1}{2}$

Hence $\log_7\sqrt{7} = \frac{1}{2}$ Ans

ii) $\log_8\frac{1}{2}$

Let $\log_8\left(\frac{1}{2}\right) = x$

$\Rightarrow 8^x = \frac{1}{2}$

$\Rightarrow \left(2^3\right)^x = 2^{-1}$

Comparing powers on both sides

$\Rightarrow 3x = -1 \quad \Rightarrow x = -\frac{1}{3}$

Hence $\log_8\left(\frac{1}{2}\right) = -\frac{1}{3}$ Ans

iii) $\log_{10}\sqrt{1000}$

Let $\log_{10}(\sqrt{1000}) = x$

$\Rightarrow 10^x = \sqrt{1000}$

$\Rightarrow 10^x = (1000)^{\frac{1}{2}} \Rightarrow 10^x = \left(10^3\right)^{\frac{1}{2}}$

$\Rightarrow 10^x = (10)^{\frac{3}{2}}$

Comparing powers on both sides

$\Rightarrow x = \frac{3}{2}$

Hence $\log_{10}\left(\sqrt{1000}\right) = \frac{3}{2}$ Ans

iv) $\log_9 3 + \log_9 27$

Given $\log_9 3 + \log_9 27 = \log_9(3\times 27)$

$\Rightarrow \log_9 3 + \log_9 27 = \log_9 81$

Let $\log_9 81 = x \Rightarrow 9^x = 81$

$\Rightarrow 9^x = 9^2$

Comparing powers on both sides

$\Rightarrow x = 2$

Hence $\log_9 3 + \log_9 27 = 2$ Ans

v) $\log\frac{1}{(0.0035)^{-4}}$

$\log\frac{1}{(0.0035)^{-4}}$ $\boxed{\because \log 1 = 0}$

$= \log 1 - \log(0.0035)^{-4}$

Hence $\log\frac{1}{(0.0035)^{-4}} = 0-(-4)\log(0.0035)$

$= 4\log(0.0035)$

$= 4(\bar{3}.5441)$ Ans

vi) $\log 45$

$\log(45) = \log(9\times 5)$

$\Rightarrow \log(45) = \log 9 + \log 5$

$\because \log mn = \log m + \log n$

$= \log 3^2 + \log 5$

$= 2\log 3 + \log 5$ Ans.

Q2: Express each of the following as a single logarithm.

i) $3\log 2 - 4\log 3$ ii) $2\log 3 + 4\log 2 - 3$

iii) $\log 5 - 1$

iv) $\frac{1}{2}\log x - 2\log 2y + 3\log z$

Solution:

i) $3\log 2 - 4\log 3$ $\because n\log x = \log x^n$

$\log m - \log n = \log\frac{m}{n}$

$\log 2^3 - \log 3^4 = \log 8 - \log 81$

$= \log\frac{8}{81}$ Ans.

ii) $2\log 3 + 4\log 2 - 3$

Given $2\log 3 + 4\log 2 - 3$

$= \log 3^2 + \log 2^4 - 3$

$= \log 9 + \log 16 - 3$

$= \log(9\times 16) - 3$

$\because \log m + \log n = \log mn$

$= \log 144 - 3$ Ans

iii) $\log 5 - 1$ $\because \log 10 = 1$

Given $\log 5 - 1$

$= \log 5 - 1\log 10 = \log\frac{5}{10}$

$= \log\frac{1}{2} = \log 0.5$ Ans

iv) $\frac{1}{2}\log x - 2\log 2y + 3\log z$

Given $\frac{1}{2}\log x - 2\log 2y + 3\log z$

$= \log x^{1/2} - \log 2^2 y^2 + \log z^3$

$= \log x^{1/2} + \log z^3 - \log 4y^2$

$= \log\frac{\sqrt{x}z^3}{4y^2}$ Ans.

Q3: Find the value of "a" from the following equations:

i) $\log_2 6 + \log_2 7 = \log_2 a$

ii) $\log_{\sqrt{3}} a = \log_{\sqrt{3}} 5 + \log_{\sqrt{3}} 8 - \log_{\sqrt{3}} 2$

iii) $\frac{\log_7 r}{\log_7 t} = \log_a r$

Solution:

i) $\log_2 6 + \log_2 7 = \log_2 a$

$\log_2 6\times 7 = \log_2 a$

$\Rightarrow \log_2 42 = \log_2 a$

Comparing the powers, we get

$\Rightarrow 42 = a$ or $\boxed{a = 42}$ Ans.

ii) $\log_{\sqrt{3}} a = \log_{\sqrt{3}} 5 + \log_{\sqrt{3}} 8 - \log_{\sqrt{3}} 2$

$\log_{\sqrt{3}} a = \log_{\sqrt{3}} 5\times 8 - \log_{\sqrt{3}} 2$

$\log_{\sqrt{3}} a = \log_{\sqrt{3}} 40 - \log_{\sqrt{3}} 2$

$\log_{\sqrt{3}} a = \log_{\sqrt{3}}\frac{40}{2} = \log_{\sqrt{3}} 20$

$\log_{\sqrt{3}} a = \log_{\sqrt{3}} 20$

$\text{Antilog}\left(\log_{\sqrt{3}} a\right) = \text{antilog}\left(\log_{\sqrt{3}} 20\right)$

$\Rightarrow \boxed{a = 20}$ Ans.

iii) $\frac{\log_7 r}{\log_7 t} = \log_a r$

Given $\frac{\log_7 r}{\log_7 t} = \log_a r$

$\log_t r = \log_a r$

$\text{Antilog}(\log_t r) = \text{antilog}(\log_a r)$

$\because \log_k a = \log_b a$

$\Rightarrow t = a$ or $\boxed{a = t}$ Ans

iv) $\log_6 25 - \log_6 5 = \log_6 a$

$\log_6\frac{25}{5} = \log_6 a$

$\Rightarrow \log_6 5 = \log_6 a$

$\text{Antilog}(\log_6 5) = \text{antilog}(\log_6 a)$

$\Rightarrow 5 = a$ or $\boxed{a = 5}$ Ans

Q4: Find $\log_2 3 \,.\, \log_3 4 \,.\, \log_4 5 \,.\, \log_5 6 \,.\, \log_6 7 \,.\, \log_7 8$

Solution:

$\log_2 3.\log_3 4.\log_4 5.\log_5 6.\log_6 7.\log_7 8$

$= \log_2 4 \log_4 6 \log_6 8$

$= \log_2 (2)^2 \log_4 8$

$= 2\log_2 2\log_4(4\times 2)$

$= 2(1)[\log_4 4 + \log_4 2]$

$= 2[1+\log_4 2] \longrightarrow (1)$

Let $\log_4 2 = x$

$\Rightarrow (4)^x = 2$

$\Rightarrow (2^2)^x = 2 \Rightarrow (2)^{2x} = 2^1$

$\Rightarrow 2x = 1 \Rightarrow x = \frac{1}{2}$

As $\log_4 2 = x \Rightarrow \log_4 2 = \frac{1}{2}$

Put this value in equation (1)

$= 2\left[1+\frac{1}{2}\right] = \cancel{2}\left[\frac{2+1}{\cancel{2}}\right]$

$= \boxed{3}$ Ans

EXAMPLE 15

Simplify (238.2) (9.506) with the help of logarithm.

Solution:

Let $x = (238.2)(9.506)$

Taking log on both sides

$\log x = \log(238.2)(9.506)$

$\Rightarrow \log x = \log(238.2) + \log(9.506)$

$\because \log mn = \log m + \log n$

$\Rightarrow \log x = 2.3770 + 0.9786 = 3.3550$

Taking antilog on both sides

$\text{Antilog}(\log x) = \text{antilog}(3.3550)$

$\boxed{x = 2265}$ Ans

EXAMPLE 16

Simplify $\frac{2.83}{(6.52)^2}$ with the help of logarithm.

Solution:

Let $x = \frac{2.83}{(6.52)^2}$

Taking log on both sides

$\log x = \log\frac{2.83}{(6.52)^2}$

$= \log 2.83 - \log(6.52)^2$

$\because \log\frac{m}{n} = \log m - \log n$

$= \log 2.83 - 2\log 6.52$

$\because \log m^n = n\log m$

$= 0.4518 - 2(0.8142)$

$= 0.4518 - 1.6284$

$= -1.766$

Add and subtract 2 to make the number positive,

$\log x = -1.1766 + 2 - 2$

$= 0.8234 - 2$

$= \bar{2}.8234$

Taking antilog on both sides

$\text{Antilog}(\log x) = \text{antilog}\,\bar{2}.8234$

$x = 0.06659$

EXERCISE 3.6

Q1: Simplify with the help of logarithm.

i) $\mathbf{3.81\times 43.4}$

ii) $\mathbf{73.42\times 0.00462\times 0.5143}$

iii) $\mathbf{\frac{784.6\times 0.0431}{28.23}}$ **iv)** $\mathbf{\frac{0.4932\times 653.7}{0.07213\times 8456}}$

v) $\mathbf{\frac{(78.41)^3\sqrt{142.3}}{\sqrt[4]{0.1562}}}$

Solution:

i) $\mathbf{3.81\times 43.4}$

Let $x = 3.81\times 43.4$

Taking log on both sides

$\log x = \log(3.81)(43.4)$

$= \log 3.81 + \log 43.4$

$= 0.5809 + 1.3749 = 2.2184$

Characteristic $= 2$

Mantissa $= 0.2184$

$x = \text{antilog}(2.2184)$

We see 0.21 in 8 column of antilogarithm table which is 1652 and see difference column 4 which is 2. Now add these, so we get 1654.

$\Rightarrow \boxed{x = 165.4}$ Ans.

ii) $x = 73.42 \times 0.00462 \times 0.5143$

Let $x = 73.42 \times 0.00462 \times 0.5143$

Taking log on both sides

$\log x = \log(73.42)(0.00462)(0.5143)$

Use law of log

$= \log(73.42) + \log(0.00462) + \log(0.5143)$

$\Rightarrow \log x = 1.8658 + \bar{3}.6646 + \bar{1}.7113$

$\Rightarrow \log x = 1.8658 - 3 + 0.6646 - 1 + 0.7113$

$\Rightarrow \log x = -0.7583$

$\Rightarrow \log x = -1 + 1 - 0.7583$

$\Rightarrow \log x = -1 + 0.2417$

$\Rightarrow \log = \bar{1}.2417$

Here characteristic $= \bar{1}$

And mantissa $= 0.2417$

Then we take antilog

We see 0.24 in 1 column of antilogarithm table which is 1742 and see in difference column 7 which is 3. Now add these which is 1745. Then

Antilog(log$x$) = antilog ($\bar{1}$.2417)

$\Rightarrow x = 0.1745$ Ans

iii) $\dfrac{784.6 \times 0.0431}{28.23}$

Let $x = \dfrac{784.6 \times 0.0431}{28.23}$

Taking log of both sides

$\log x = \log\left(\dfrac{784.6 \times 0.0431}{28.23}\right)$

Use law of log

$= \log(784.6) + \log(0.0431) - \log(28.23)$

$\Rightarrow \log x = 2.8946 + \bar{2}.6345 - 1.4507$

$\Rightarrow \log x = 2.8946 - 2 + 0.6345 - 1.4507$

$\Rightarrow \log x = 0.0784$

Here characteristic $= 0$

And mantissa $= 0.0784$

We see 0.07 in column 8 of antilogarithm table which is 1197 and see in difference column 4 which is 1. Now add these which result 1198. Then

Antilog (log x) = antilog (0.0784)

$\Rightarrow x = 1.1980$ Ans

iv) $\dfrac{0.4932 \times 653.7}{0.07213 \times 8456}$

Let $x = \dfrac{0.4932 \times 653.7}{0.07213 \times 8456}$

Taking log of both sides

$\log x = \log\left(\dfrac{0.4932 \times 653.7}{0.07213 \times 8456}\right)$

Use laws of log

$= \log(0.4932) + \log(653.7) - \log(0.07213)$
$- \log(8456)$

$\Rightarrow \log x = \bar{1}.6930 + 2.8154 - \bar{2}.8581 - 3.9272$

$\Rightarrow \log x = -1 + 0.6930 + 2.8154 - (-2 + 0.8581)$
-3.9272

$\Rightarrow \log x = -1 + 0.6930 + 2.8154 + 2$
$-0.8581 - 3.9272$

$\Rightarrow \log x = -0.2769$

$\Rightarrow \log x = -1 + 1 - 0.2769 = -1 + 0.7231$

$\Rightarrow \log x = \bar{1}.7231$

Here characteristic $= \bar{1}$

And mantissa $= 0.7231$

Then we take antilog

We see 0.72 in column 3 of antilogarithm table which is 5284 and see in difference column 1 which is 1. Now add these which result 5285. Then

Antilog (log x) = antilog ($\bar{1}$.7231)

$\Rightarrow x = 0.5285$

v) $\dfrac{(78.41)^3 \times \sqrt{142.3}}{\sqrt[4]{0.1562}}$

Let $x = \dfrac{(78.41)^3 \times \sqrt{142.3}}{\sqrt[4]{0.1562}}$

Taking log of both sides

$\log x = \log\left(\dfrac{(78.41)^3 \times \sqrt{142.3}}{\sqrt[4]{0.1562}}\right)$

Use laws of log

$= \log(78.41)^3 + \log(142.3)^{\frac{1}{2}} - \log\left(\sqrt[4]{0.1562}\right)$

$= 3\log(78.41) + \dfrac{1}{2}.\log(142.3) - \log(0.1562)^{\frac{1}{4}}$

$\Rightarrow \log x = 3\log(78.41) + \frac{1}{2}.\log(142.3)$

$-\frac{1}{4}(\bar{1}.1937)$

$\Rightarrow \log x = 5.6832 + 1.0766 - \frac{1}{4}(-1+0.1937)$

$\Rightarrow \log x = 6.7598 - (-0.2016)$

$\Rightarrow \log x = 6.7598 + 0.2016$

$\Rightarrow \log x = 6.9614$

Here characteristic = 6

And mantissa = 0.9614

Then we take antilog

We see 0.96 in column 1 of antilogarithm table which is 9141 and see in difference column 4 which is 8. Now add these which result 9149. Then

$\text{Antilog}(\log x) = \text{antilog}(6.9614)$

$\Rightarrow x = 9149000$

Q2: Find the following if $\log 2 = 0.3010$, $\log 3 = 0.4771$, $\log 5 = 0.6990$, $\log 7 = 0.8451$.

i) log 105 **ii) log 108**

iii) $\log\sqrt[3]{72}$ **iv) log 2.4**

v) log 0.0081

Solution:

i) log 105

$\log 105 = \log 5\times 7\times 3$

Use laws of log.

$= \log 5 + \log 7 + \log 3$

Putting the values

$= 0.6990 + 0.8451 + 0.4771$

$= 2.0212$ Ans.

ii) log 108

$\log 108 = \log 4\times 27 = \log 4 + \log 27$

$= \log 2^2 + \log 3^3 = 2\log 2 + 3\log 3$

Putting the values

$= 2(0.3010) + 3(0.4771)$

$= 0.602 + 1.4313$

$= 2.0333$ Ans.

iii) $\log\sqrt[3]{72}$

$\log\sqrt[3]{72} = \log(72)^{1/3} = \frac{1}{3}\log(72)$

Use laws of log

$= \frac{1}{3}\log 72 = \frac{1}{3}\log 9\times 8$

$= \frac{1}{3}[\log 9 + \log 8] = \frac{1}{3}\left[\log 3^2 + \log 2^3\right]$

$= \frac{1}{3}[2\log 3 + 3\log 2]$

Putting the values

$= \frac{1}{3}[2(0.4771) + 3(0.3010)]$

$= \frac{1}{3}[0.9542 + 0.903] = \frac{1}{3}(1.8572)$

$= 0.619066$ Ans.

iv) log 2.4

$\log 2.4 = \log\frac{24}{10} = \log 24 - \log 10$

$= \log 8\times 3 - \log 2\times 5$

$= \log 8 + \log 3 - (\log 2 + \log 5)$

$= \log 2^3 + \log 3 - \log 2 - \log 5$

$= 3\log 2 + \log 3 - \log 2 - \log 5$

Putting the values

$= 3(0.3010) + 0.4771 - 0.3010 - 0.6990$

$= 0.903 + 0.4771 - 0.3010 - 0.6990$

$= 0.3801$ Ans.

v) log 0.0081

$\log 0.0081 = \log\frac{81}{10000}$

$= \log 81 - \log 10000$

$= \log 3^4 - \log 10^4$

$= 4\log 3 - 4(\log 2\times 5)$

$= 4\log 3 - 4[\log 2 + \log 5]$

Putting the values

$= 4(0.4771) - 4(0.3010 + 0.6990)$

$= 1.9084 - 4$

$= -2.0916$ Ans.

Review Exercise 3

Q1: Select the correct answer.

i) $\log_9 \frac{1}{81}$

(a) -1 ✓(b) -2
(c) 2 (d) does not exist

ii) If $\log_2 8 = x$ then $x =$

(a) 64 (b) 32
✓(c) 3 (d) 28

iii) Base of common log is:

✓(a) 10 (b) e
(c) π (d) 5

iv) $\log\sqrt{10} =$

(a) -1 (b) $-\frac{1}{2}$
✓(c) $\frac{1}{2}$ (d) 2

v) For any non-zero value of $x, x^0 =$

(a) 2 ✓(b) 1
(c) 0 (d) 10

vi) Rewrite $t = \log_b m$ as an exponential equation:

(a) $t = m^b$ (b) $b^m = t$
✓(c) $m = b^t$ (d) $m^t = b$

vii) $\log_{10} 10 =$

(a) 2 (b) 3
(c) 0 ✓(d) 1

viii) Characteristic of log 0.000059 is:

✓(a) -5 (b) 5
(c) -4 (d) 4

ix) Evaluate $\log_7 \frac{1}{\sqrt{7}}$

(a) -1 ✓(b) $-\frac{1}{2}$
(c) $\frac{1}{2}$ (d) 1

x) Base of natural log is:

(a) 10 ✓(b) e
(c) -1 (d) not defined

xi) $\log m + \log n =$

(a) $\log m \log n$ (b) $\log m - \log n$
✓(c) $\log mn$ (d) $\log \frac{m}{n}$

xii) 0.069 can be written in scientific notation as:

(a) 6.9×10^{1} ✓(b) 6.9×10^{-2}
(c) 0.69×10^{1} (d) 69×10^{2}

xiii) $\ln x - 2\ln y$

(a) $\ln\frac{x}{y}$ (b) $\ln xy^2$
(c) $\ln\frac{x^2}{y}$ ✓(d) $\ln\frac{x}{y^2}$

Q2: Write 9473.2 in scientific notation.

Solution:

Given 9473.2

$9473.2 \Rightarrow 9.473\times10^{3}$ Ans.

which is in scientific notation.

Q3: Write 5.4×10^6 in standard notation.

Solution:

Given $5.4\times10^6 \Rightarrow \frac{54}{10}\times10^6$

$= \frac{54}{1\cancel{0}}\times100000\cancel{0}$

$= 5400000$ Ans.

which is in standard notation.

Q4: Write in logarithmic form: $3^{-3} = \frac{1}{27}$.

Solution:

As $a^y = x \Rightarrow \log_a x = y$

As $3^{-3} = \frac{1}{27} \Rightarrow \log_3 \frac{1}{27} = -3$ Ans.

Q5: Write in exponential form: $\log_5 1 = 0$.

Solution:

Given $\log_5 1 = 0$

As $\log_a x = y \Rightarrow a^y = x$

$\therefore \log_5 1 = 0 \Rightarrow \boxed{5^0 = 1}$ Ans.

Q6: Solve for $x : \log_4 16 = x$

Solution:

As $\log_a x = y \Rightarrow a^y = x$

$\therefore \log_4 16 = x \Rightarrow (4)^x = 16$

$\Rightarrow (4)^x = (4)^2 \Rightarrow \boxed{x = 2}$ Ans.

Q7: Find the characteristic of the common logarithm 0.0083.

Solution:

Given 0.0083 in scientific notation

$=8.3\times10^{-3}$

Hence characteristic $=-3$ Ans.

Q8: Find log 12.4

Solution:

Given log 12.4

In scientific notation

$12.4=1.24\times10^{1}$

Here characteristic $=1$

Foe mantissa we see 12 in column 4 in log table which is 0934.

Hence $\log 12.4=1.0934$ Ans.

Q9: Find the value of 'a',

$\log_{\sqrt{5}}3a=\log_{\sqrt{5}}9+\log_{\sqrt{5}}2-\log_{\sqrt{5}}3$

Solution:

Given $\log_{\sqrt{5}}3a=\log_{\sqrt{5}}9+\log_{\sqrt{5}}2-\log_{\sqrt{3}}2$

$\log_{\sqrt{5}}3a=\log_{\sqrt{5}}\frac{(\cancel{9}^{3}\times2)}{\cancel{3}}$

$\log_{\sqrt{5}}3a=\log_{\sqrt{5}}(3\times2)=\log_{\sqrt{5}}6$

$\log_{\sqrt{5}}3a=\log_{\sqrt{5}}6$

Taking anti-log

Antilog$(\log_{\sqrt{5}}3a)=$ antilog$(\log_{\sqrt{5}}6)$

$\Rightarrow 3a=6\Rightarrow a=\frac{6}{3}$

$\Rightarrow \boxed{a=2}$ Ans.

THINK:

Q10: $\frac{(63.28)^{3}.(0.00843)^{2}(0.4623)}{(412.3)(2.184)^{5}}$

Solution:

Let $x=\frac{(63.28)^{3}.(0.00843)^{2}(0.4623)}{(412.3)(2.184)^{5}}$

Taking log of both sides

$\log x=\log\frac{(63.28)^{3}.(0.00843)^{2}(0.4623)}{(412.3)(2.184)^{5}}$

Use logarithmic laws

$=\log(63.28)^{3}+\log(0.00843)^{2}+\log(0.4623)$

$-\log(412.3)-\log(2.184)^{5}$

$\log x=3\log(63.28)+2\log(0.00843)+\log(0.4623)$

$-\log(412.3)-5\log(2.184)$

$\Rightarrow \log x=3(1..8012)+2(\bar{3}.9258)+\bar{1}.6649$

$-2.6152-5(0.3393)$

$\Rightarrow \log x=5.5221+2(-3+0.9258)-1+0.6649$

$-2.6152\ -1.6965$

$\Rightarrow \log x=0.7592+2(-2.0642)$

$\Rightarrow \log x=0.7592-4.1284$

$\Rightarrow \log x=-3.3692$

$\Rightarrow \log x=-4+4-3.3692$

$\Rightarrow \log x=-4+0..6308$

$\Rightarrow \log x=\bar{4}.6308$

Here characteristic $=\bar{4}$

And mantissa $=0.6308$

Taking antilog on both sides

Antilog$(\log x)=$ antilog$(\log\bar{4}.6308)$

We see 0.63 in column 0 of antilogarithm table which is 4266 and see in difference column 8 which is 8. Now add these which result $4266+8=4274$. Then

Antilog$(\log x)=$ antilog$(\bar{4}.6308)$

$\Rightarrow x=0..0004274$

Or $\boxed{x=0.0004}$ Ans.

***

Additional MCQs of Unit 3:
Logarithm

1. $2\log_a m + 3\log_a n =$ ....
   (a) mn (b) m^2n^2 (c) $\log_a m^2n^3$ (d) none
   ✓ **Ans. (c) $\log_a m^2n^2$**
2. $4\log_3 3 =$ ....
   (a) 1 (b) 4 (c) 81 (d) 16
   ✓ **Ans. (a) 1**
3. $\log_b a =$ .... (a) $\dfrac{1}{-\log_b a}$ (b) $\dfrac{1}{\log_a b}$ (c) $\dfrac{1}{(\log_b a)^{-1}}$
   ✓ **Ans. (b) $\dfrac{1}{\log_a b}$**
4. $\log_{10} x$ is..........log.
   (a) Natural (b) Common (c) General (d) none
   ✓ **Ans. (c) Common**
5. In scientific notation $0.00729 =$ ....
   (a) 7.29 (b) 729×10^3 (c) 7.29×10^{-3} (d) none
   ✓ **Ans. (c) 7.29×10^{-3}**
6. The exponential form of $\sqrt[3]{16} =$ ....
   (a) 2^3 (b) $2^{4/3}$ (c) 16 (d) 4
   ✓ **Ans. (b) $2^{4/3}$**
7. The radical form of $(2x+5)^{2/3} =$ ....
   (a) $\sqrt{2x+5}$ (b) $\sqrt[3]{2x+5}$ (c) $\sqrt[3]{(2x+5)^2}$ (d) none
   ✓ **Ans. (c) $\sqrt[3]{(2x+5)^2}$**
8. log 2345 has characteristic..........
   (a) 2 (b) 3 (c) 4 (d) $\bar{3}$
   ✓ **Ans. (b) 3**
9. $\log_{30} 2 + \log_{30} 3 + \log_{30} 5 =$ ....
   (a) $\log_{30} 15$ (b) 2 (c) 30 (d) 1
   ✓ **Ans. (d) 1**
10. $\log_5 \sqrt{25} =$ ....
   (a) 1 (b) 2 (c) 5 (d) 25
   ✓ **Ans. (a) 1**

UNIT 4:
ALGEBRAIC EXPRESSIONS & ALGEBRAIC FORMULAS

Algebraic Expression:
That expression which consists of variables and constants with operators is known as *algebraic expression*. For example, $3x+5$, x^2+2x+3

Rational Expression:

An algebraic expression of the form $\frac{p(x)}{q(x)}$ where $p(x), q(x)$ are polynomials and $q(x) \neq 0$ is called a *rational expression*. For example,

i) $\frac{x^2-6x+1}{x+7}$ if $x+7\neq 0$

ii) $\frac{x^2+4x+5}{3}$ is a rational expression.

Every polynomial is a rational expression.

How to examine whether a given algebraic expression is: a) Polynomial or not b) Rational expression or not?

Every algebraic expression may not be a polynomial. For example

$3x^2+6x+\frac{7}{x}$ with $x\neq 0$ is an algebraic expression but it is not a polynomial.

We know that a rational expression is always in the form $\frac{p(x)}{q(x)}$. Now $\sqrt{\frac{3x^2-5x-2}{10-x}}$ is an algebraic expression with $x\neq 10$ but it is not a rational expression.

Lowest terms of a Rational Expression $p(x)/q(x)$:

A rational expression $\frac{p(x)}{q(x)}$ is said to be in its lowest term if *p(x)* and *q(x)* are polynomials with integral coefficients and having no common factor. For example $\frac{6}{x^2-4x}$ is in its lowest term while $\frac{4x+20}{(x+5)(x+1)(x-8)}$ is not.

EXAMPLE 1

Examine whether the following rational expressions are in their lowest term or not: i) $\frac{9}{x-3}$ ii) $\frac{x^2-7x}{x+4}$ iii) $\frac{x^2-1}{x-1}$

Solution:

i) $\frac{9}{x-3}$ is in its lowest terms, because it has no factor in common in the numerator and denominator.

ii) $\frac{x^2-7x}{x+4}$ is in its lowest terms, because it has no factor in common in the numerator and denominator.

iii) $\frac{x^2-1}{x-1}$ is not in its lowest terms as $(x-1)$ is a common factor in the numerator and denominator, because we can write it as

$$\frac{x^2-1}{x-1}=\frac{(x-1)(x+1)}{x-1}$$

EXAMPLE 2

Reduce the following rational expressions to their lowest terms:

i) $\frac{x+3}{(x+3)(x-2)}$ ii) $\frac{x^2-b^2}{(x+b)x^2}$

Solution:

i) $\frac{x+3}{(x+3)(x-2)}$

$=\frac{1}{x-2}$ (Cancelling common factors)

ii) $\frac{x^2-b^2}{(x+b)x^2}$

$=\frac{(x+b)(x-b)}{(x+b)x^2}=\frac{x-b}{x^2}$

Which is in lowest terms.

EXAMPLE 3

Add $\dfrac{x+2}{x+3}$ and $\dfrac{x+5}{x+3}$.

Solution:

Here the denominator in both the rational expressions is the same, so LCM is $x+3$.

$$\therefore \quad \frac{x+2}{x+3}+\frac{x+5}{x+3}=\frac{x+2+x+5}{x+3}=\frac{2x+7}{x+3}$$

EXAMPLE 4

Add $\dfrac{4}{x-3}$ and $\dfrac{6}{x+3}$.

Solution:

$$\frac{4}{x-3}+\frac{6}{x+3}=\frac{4(x+3)+6(x-3)}{(x-3)(x+3)}$$

$\because$ LCM of $x-3$ & $x+3$ is $(x-3)(x+3)$

$$=\frac{4x+12+6x-18}{x^2-9}=\frac{10x-6}{x^2-9}$$

EXERCISE 4.1

Q1: Which of the following expressions are polynomials:

i) $1-5y+8y^2+6y^3$

ii) $\dfrac{5}{x^2}+\dfrac{3}{4x+1}$ iii) $\dfrac{\sqrt{x}}{6x-1}$

Solution:

i) $1-5y+8y^2+6y^3$

This rational expression is a polynomial.

ii) $\dfrac{5}{x^2}+\dfrac{3}{4x+1}$

This rational expression is not a polynomial.

iii) $\dfrac{\sqrt{x}}{6x-1}$

This rational expression is not a polynomial.

Q2: Which of the following rational expressions are in their lowest terms?

i) $\dfrac{5y^2-5}{y-1}$ ii) $\dfrac{x^2-9}{x-2}$

iii) $\dfrac{x+y}{x^2-y^2}$

Solution:

i) $\dfrac{5y^2-5}{y-1}$

This expression is in lowest term due to common factors.

ii) $\dfrac{x^2-9}{x-2}$

This expression is in lowest terms.

iii) $\dfrac{x+y}{x^2-y^2}$

This expression is not in lowest terms.

Q3: Reduce the following rational expressions to their lowest terms:

i) $\dfrac{x-5}{x^2-5x}$ ii) $\dfrac{t^3(t-3)}{(t-3)(t+5)}$

iii) $\dfrac{x^4+\frac{1}{x^4}}{x^2-\frac{1}{x^2}}$ iv) $\dfrac{2a+6}{a^2-9}$

Solution:

i) $\dfrac{x-5}{x^2-5x}$

Given $\dfrac{x-5}{x^2-5x}$

$$=\frac{\cancel{(x-5)}}{x\cancel{(x-5)}}=\frac{1}{x} \quad \text{Ans.}$$

$\dfrac{t^3(t-3)}{(t-3)(t+5)}$

Given $\dfrac{t^3(t-3)}{(t-3)(t-5)}$

$$=\frac{t^3\cancel{(t-3)}}{\cancel{(t-3)}(t+5)}=\frac{t^3}{t+5} \quad \text{Ans.}$$

iii) $\dfrac{x^4+\dfrac{1}{x^4}}{x^2-\dfrac{1}{x^2}}$

Given $\dfrac{x^4+\dfrac{1}{x^4}}{x^2-\dfrac{1}{x^2}}$

$=\dfrac{x^4+\dfrac{1}{x^4}}{\left(x-\dfrac{1}{x}\right)\left(x+\dfrac{1}{x}\right)}$ Ans.

iv) $\dfrac{2a+6}{a^2-9}$

Given $\dfrac{2a+6}{a^2-9}=\dfrac{2(a+3)}{(a)^2-(3)^2}$

$=\dfrac{2\cancel{(a+3)}}{\cancel{(a+3)}(a-3)}$

$=\dfrac{2}{a-3}$ Ans.

Q4: Add the following rational expressions:

i) $4x^2-5x-10,\ 2x^2+5x+10$

ii) $\dfrac{y+9}{y^2+3},\dfrac{-7y+7}{y^2+3}$

iii) $\dfrac{y}{y+4},\dfrac{2y}{y-4}$

iv) $\dfrac{t}{t^2-25},\dfrac{3t}{t+5}$

Solution:

i) $4x^2-5x-10,\ 2x^2+5x+10$

Given $4x^2-5x-10,\ 2x^2+5x+10$

Adding both expressions, we get

$$\begin{array}{l} 4x^2-\cancel{5x}-\cancel{10} \\ \underline{+2x^2+\cancel{5x}+\cancel{10}} \\ =6x^2 \end{array}$$ Ans.

ii) $\dfrac{y+9}{y^2+3},\dfrac{-7y+7}{y^2+3}$

Given $\dfrac{y+9}{y^2+3},\dfrac{-7y+7}{y^2+3}$

Adding both expressions, we get

$\dfrac{(y+9)}{y^2+3}+\dfrac{(-7y+7)}{y^2+3}$

$=\dfrac{y+9-7y+7}{y^2+3}$

$=\dfrac{-6y+16}{y^2+3}$ Ans.

iii) $\dfrac{y}{y+4},\dfrac{2y}{y-4}$

Given $\dfrac{y}{y+4},\dfrac{2y}{y-4}$

Adding both expressions, we get

$\dfrac{y}{y+4}+\dfrac{2y}{y-4}$

$=\dfrac{y(y-4)+2y(y+4)}{(y+4)(y-4)}$

$=\dfrac{y^2-4y+2y^2+8y}{(y)^2-(4)^2}$

$=\dfrac{3y^2+4y}{y^2-16}$ Ans.

iv) $\dfrac{t}{t^2-25},\dfrac{3t}{t+5}$

Given $\dfrac{t}{t^2-25},\dfrac{3t}{t+5}$

$=\dfrac{t}{(t)^2-(5)^2},\dfrac{3t}{t+5}$

$\dfrac{t}{(t+5)(t-5)},\dfrac{3t}{t+5}$

Adding both expressions, we get

$=\dfrac{t}{(t+5)(t-5)}+\dfrac{3t}{t+5}$

$=\dfrac{t+3t(t-5)}{(t+5)(t-5)}$

$$=\frac{t+3t^2-15t}{t^2-25}$$

$$=\frac{3t^2-14t}{t^2-25} \quad \text{Ans.}$$

Q5: Subtract the first expression from the second in the following:

i) $y^2+4y-15$, $8y^2+2$

ii) $\frac{8x^2-7}{x^2+1}, \frac{8x^2+7}{x^2+1}$

iii) $\frac{1}{a-3}, \frac{2a}{a^2-9}$ iv) $\frac{x}{3x-6}, \frac{x+2}{x-2}$

Solution:

i) $y^2+4y-15$, $8y^2+2$

Given $y^2+4y-15, 8y^2+2$

$$=(8y^2+2)-(y^2+4y-15)$$

$$=8y^2+2-y^2-4y+15$$

$$=7y^2-4y+17 \quad \text{Ans.}$$

ii) $\frac{8x^2-7}{x^2+1}, \frac{8x^2+7}{x^2+1}$

Subtracting 1st from 2nd expression

$$\frac{8x^2+7}{x^2+1}-\frac{8x^2-7}{x^2+1}$$

$$=\frac{\cancel{8x^2}+7-\cancel{8x^2}+7}{x^2+1}$$

$$=\frac{14}{x^2+1} \quad \text{Ans.}$$

iii) $\frac{1}{a-3}, \frac{2a}{a^2-9}$

Given $\frac{1}{a-3}, \frac{2a}{(a)^2-(3)^2}$

$$=\frac{1}{a-3}, \frac{2a}{(a+3)(a-3)}$$

Subtracting first expression from 2nd expression

$$\because a^2-b^2=(a-b)(a+b)$$

$$=\frac{2a}{(a+3)(a-3)}-\frac{1}{(a-3)}$$

$$=\frac{2a-(a+3)}{(a+3)(a-3)}=\frac{2a-a-3}{a^2-9}$$

$$=\frac{a-3}{a^2-9}=\frac{\cancel{a-3}}{(a+3)\cancel{(a-3)}}$$

$$=\frac{1}{a+3} \quad \text{Ans.}$$

iv) $\frac{x}{3x-6}, \frac{x+2}{x-2}$

Given $\frac{x}{3x-6}, \frac{x+2}{x-2}$

$$=\frac{x}{3(x-2)}, \frac{x+2}{x-2}$$

Subtracting first from 2nd expression

$$=\frac{x+2}{x-2}-\frac{x}{3(x-2)}$$

$$=\frac{3(x+2)-x}{3(x-2)}=\frac{3x+6-x}{3(x-2)}$$

$$=\frac{2x+6}{3(x-2)}=\frac{2(x+3)}{3(x-3)} \quad \text{Ans.}$$

Q6: Simplify the following:

i) $\frac{2x}{6x-9} \cdot \frac{4x-6}{x^2+x}$

ii) $\frac{x+4}{3-x} \cdot \frac{x^2-9}{x^2-16}$

iii) $\frac{3x-15}{2x+6} \cdot \frac{x^2-9}{x^2-25}$

Solution:

i) $\frac{2x}{6x-9} \cdot \frac{4x-6}{x^2+x}$

$$=\frac{2x}{3(2x-3)}\times\frac{2(2x-3)}{x(x+1)}$$

$$=\frac{2x}{3\cancel{(2x-3)}}\times\frac{2\cancel{(2x-3)}}{x(x+1)}$$

$$=\frac{4\cancel{x}}{3\cancel{x}(x+1)}=\frac{4}{3(x+1)} \quad \text{Ans.}$$

ii) $\frac{x+4}{3-x} \cdot \frac{x^2-9}{x^2-16}$

Given $\frac{x+4}{3-x} \cdot \frac{x^2-9}{x^2-16}$

$$\frac{(x+4)}{(3-x)}\times\frac{(x)^2-(3)^2}{(x)^2-(4)^2}$$

$$=\frac{\cancel{(x+4)}\,\cancel{(x-3)}(x+3)}{-\cancel{(x-3)}(x-4)\cancel{(x+4)}}$$

$$=\frac{x+3}{-(x-4)}=\frac{x+3}{4-x}\text{ Ans.}$$

iii) $\dfrac{3x-15}{2x+6}\cdot\dfrac{x^2-9}{x^2-25}$

Given $\dfrac{3x-15}{2x+6}\cdot\dfrac{x^2-9}{x^2-25}$

$$=\frac{3(x-5)}{2(x+3)}\times\frac{(x)^2-(3)^2}{(x)^2-(5)^2}$$

$$=\frac{3\cancel{(x-5)}\,\cancel{(x+3)}(x-3)}{2\cancel{(x+3)}(x+5)\cancel{(x-5)}}$$

$$=\frac{3(x-3)}{2(x+5)}\quad\text{Ans.}$$

Q7: Simplify the following:

i) $\dfrac{2y-10}{3y}\div(y-5)$

ii) $\dfrac{p}{q}\div\dfrac{r}{q}\cdot\dfrac{p}{q}$

iii) $\dfrac{a^2-9}{(a-6)(a+4)}\div\dfrac{a-3}{a-6}$

Solution:

i) $\dfrac{2y-10}{3y}\div(y-5)$

Given $\dfrac{2y-10}{3y}\div(y-5)$

$$=\frac{2\cancel{(y-5)}}{3y}\times\frac{1}{\cancel{(y-5)}}$$

$$=\frac{2}{3y}\quad\text{Ans.}$$

ii) $\dfrac{p}{q}\div\dfrac{r}{q}\cdot\dfrac{p}{q}$

Given $\dfrac{p}{q}\div\dfrac{r}{q}\cdot\dfrac{p}{q}$

$$=\frac{p}{\cancel{q}}\times\frac{\cancel{q}}{r}\cdot\frac{p}{q}$$

$$=p\times\frac{1}{r}\times\frac{p}{q}$$

$$=\frac{p\times1\times p}{r\times q}$$

$$=\frac{p^2}{rq}\quad\text{Ans.}$$

iii) $\dfrac{a^2-9}{(a-6)(a+4)}\div\dfrac{a-3}{a-6}$

Given $\dfrac{a^2-9}{(a-6)(a+4)}\div\dfrac{a-3}{a-6}$

$$=\frac{(a)^2-(3)^2}{(a-6)(a+4)}\div\frac{a-3}{a-6}$$

$$\because a^2-b^2=(a-b)(a+b)$$

$$=\frac{(a+3)\cancel{(a-3)}}{\cancel{(a-6)}(a+4)}\times\frac{\cancel{(a-6)}}{\cancel{(a-3)}}$$

$$=\frac{a+3}{a+4}\quad\text{Ans.}$$

<u>Value of an Algebraic Expression at Some Particular Real Number</u>:

If we are given an algebraic expression which is a polynomial in x. Then we replace x in $p(x)$ by the given real number. For example if $p(x)=3x^2+4x-9$ is a given polynomial and we have to find its value at $x=3$ then

$$p(3)=3(3)^2+4(3)-9$$
$$=3(9)+12-9$$
$$=27+12-9$$
$$=39-9=30 \text{ required value of } p(x)$$

EXAMPLE 10

When $x=-3$, find the value of:

i) $2x^2-3x$ **ii)** $(3x)^2-x^2$

<u>Solution</u>:

i) $2x^2-3x=2(-3)^2-3(-3)$

By putting $x=-3$

$=2(9)+9=18+9=27$

ii) $(3x)^2-x^2=\left(3(-3)^2\right)-(-3)^2$

$=(-9)^2-9$

By putting $x=-3$

$=81-9=72$

EXAMPLE

When $x=2$, $y=-3$, evaluate the following: i) x^2-y^2 ii) $(2x)^2-(3y)^2$

Solution:

i) x^2-y^2

$=2^2-(-3)^2$

$=4-9=-5$

ii) $(2x)^2-(3y)^2$

$=(2\times 2)^2-(3\times(-3))^2$

By putting $x=2$, $y=-3$

$=4^2-(-9)^2$

$=16-81=-65$

EXERCISE 4.2

Q1: Evaluate the following when $a=3$, $b=-1$, $c=2$.

i) $5a-10$ **ii)** $3b+5c$

iii) $2a-3b+2c$

Solution:

i) $5a-10$

Given $5a-10$

Putting value of $a=3$

$5(3)-10=15-10=5$ Ans.

ii) $3b+5c$

Given $3b+5c$

Putting values of $b=-1$, $c=2$

$3(-1)+5(2)$

$=-3+10=7$ Ans.

iii) $2a-3b+2c$

Given $2a-3b+2c$

Putting the values $a=3$, $b=-1$, $c=2$

$=2(3)-3(-1)+2(2)$

$=6+3+4=13$ Ans.

Q2: Evaluate the following for $x=-5$ and $y=2$.

i) $7-3xy$ **ii)** x^2+xy+y^2

iii) $(3x)^2-(4y)^2$

Solution:

i) $7-3xy$

Given $7-3xy$

Putting the values $x=-5$, $y=2$

$=7-3(-5)(2)$

$=7+30=37$ Ans.

ii) x^2+xy+y^2

Given x^2+xy+y^2

Putting the values $x=-5$, $y=2$

$=(-5)^2+(-5)(2)+(2)^2$

$=25-10+4=19$ Ans.

iii) $(3x)^2-(4y)^2$

Given $(3x)^2-(4y)^2$

$=9x^2-16y^2$

Putting the values $x=-5$, $y=2$

$=9x^2-16y^2$

$=9(-5)^2-16(2)^2$

$=9(25)-16(4)$

$=225-64=161$ Ans.

Q3: Evaluate the following when $k=-2$, $\ell=3$, $m=4$.

i) $k^2(2\ell-3m)$ ii) $5m\sqrt{k^2+\ell^2}$

iii) $\dfrac{k+l+m}{k^2+\ell^2+m^2}$

Solution:

i) $k^2(2\ell-3m)$

Given $k^2(2\ell-3m)$

Putting the values $k=-2$, $\ell=3$ & $m=4$

$=(-2)^2\left[2(3)-3(4)\right]$

$=4(6-12)=4(-6)=-24$ Ans.

ii) $5m\sqrt{k^2+\ell^2}$

Given $5m\sqrt{k^2+\ell^2}$

Putting the values $k=-2$, $\ell=3$ & $m=4$

$= 5(4)\sqrt{(-2)^2+(3)^2}$

$= 20\sqrt{4+9}$

$= 20\sqrt{13}$ Ans.

iii) $\frac{k+l+m}{k^2+\ell^2+m^2}$

Given $\frac{k+\ell+m}{k^2+\ell^2+m^2}$

Putting the values of $k = -2$, $\ell = 3$ & m = 4 here

$= \frac{-2+3+4}{(-2)^2+(3)^2+(4)^2}$

$= \frac{5}{4+9+16} = \frac{5}{29}$ Ans.

Q4: Evaluate $\frac{a+1}{4a^2-1}$ when $a = \frac{1}{2}$ and $a = -\frac{1}{2}$.

Solution:

Given $\frac{a+1}{4a^2-1}$

$\frac{a+1}{4a^2-1}$

Putting the value $a = \frac{1}{2}$, we get

$= \frac{\frac{1}{2}+1}{4\left(\frac{1}{2}\right)^2-1}$

$= \frac{\frac{1+2}{2}}{\cancel{4}\times\frac{1}{\cancel{4}}-1} = \frac{\frac{3}{2}}{1-1}$

$= \frac{\frac{3}{2}}{0} = \infty$ OR undefined value

When $a = -\frac{1}{2}$

Put in $\frac{a+1}{4a^2-1}$

Then $\frac{\frac{-1}{2}+1}{4\left(\frac{-1}{2}\right)^2-1} = \frac{\frac{-1+2}{2}}{\cancel{4}\times\frac{1}{\cancel{4}}-1}$

$= \frac{\frac{1}{2}}{0} = \infty$

Hence the result in both cases is undefined.

Q5: If a = 9, b = 12, c = 15 and $S = \frac{a+b+c}{2}$. Find the value of $\sqrt{S(S-a)(S-b)(S-c)}$

Solution:

Given $S = \frac{a+b+c}{2}$

Putting the values $a = 9, b = 12$ & $c = 15$

$= \frac{9+12+15}{2}$

$= \frac{36}{2} = 18$

$\Rightarrow \boxed{S = 18}$

We have to find the value of

$\sqrt{S(S-a)(S-b)(S-c)}$

Putting the values $a = 9, b = 12$ & $c = 15$

$= \sqrt{18(18-9)(18-12)(18-15)}$

$= \sqrt{18\times9\times6\times3}$

$= \sqrt{3\times3\times2\times3\times3\times3\times2\times3}$

$= 3\times3\times3\times2 = 54$ Ans.

Algebraic Formulas:

Establish the formulas

a) $(a+b)^2+(a-b)^2 = 2(a^2+b^2)$

b) $(a+b)^2-(a-b)^2 = 4ab$

c) $(a+b)^2 = a^2+2ab+b^2 \rightarrow (1)$

d) $(a-b)^2 = a^2-2ab+b^2 \rightarrow (2)$

e) $a^2-b^2 = (a+b)(a-b) \rightarrow (3)$

Proof:

a) Adding (1) and (2)

$(a+b)^2+(a-b)^2$

$=a^2+\cancel{2ab}+b^2+a^2-\cancel{2ab}+b^2$
$=2a^2+2b^2=2(a^2+b^2)$

Here $\boxed{(a+b)^2+(a-b)^2=2(a^2+b^2)}$ proved.

b) Subtracting (1) and (2)
$(a+b)^2-(a-b)^2$
$=(a^2+b^2)+2ab)-(a^2+b^2-2ab)$
$=\cancel{a^2}+2ab+\cancel{b^2}-\cancel{a^2}+2ab-\cancel{b^2}$
$=4ab$

EXAMPLE

Find the values a^2+b^2 and ab, when $a+b=5$, $a-b=2$.

Solution:
$a+b=5\Rightarrow(a+b)^2=5^2$
$\Rightarrow\quad a^2+2ab+b^2=25\longrightarrow(1)$
$a-b=2\Rightarrow(a-b)^2=2^2$
$\Rightarrow\quad a^2-2ab+b^2=4\longrightarrow(2)$
Adding equations (1) and (2), we get
$(a^2+2ab+b^2)+(a^2-2ab+b^2)=25+4$
$2a^2+2b^2=29$ or $a^2+b^2=\dfrac{29}{2}$
From equation (1)
$2ab=25-(a^2+b^2)=25-\dfrac{29}{2}$
Putting $a^2+b^2=\dfrac{29}{2}$
$2ab=16$ or $ab=8$

EXERCISE 4.3

Q1: Find the values of x^2+y^2 and xy, when:

i) $x+y=8,\ x-y=3$
ii) $x+y=10,\ x-y=7$
iii) $x+y=11,\ x-y=5$
iv) $x+y=7,\ x-y=4$

Solution:
i) $x+y=8,\ x-y=3$
We know that from formula
$(x+y)^2+(x-y)^2=2(x^2+y^2)$
Putting values in L.H.S
$=(8)^2+(3)^2=2(x^2+y^2)$
$64+9=2(x^2+y^2)$
$73=2(x^2+y^2)$
OR $2(x^2+y^2)=73$
$\dfrac{\cancel{2}}{\cancel{2}}(x^2+y^2)=\dfrac{\cancel{73}^{36.5}}{\cancel{2}}$ (Divide by 2)
$\Rightarrow\boxed{x^2+y^2=36.5}$ Ans

Now use formula $(x+y)^2-(x-y)^2=4xy$
Putting the values in L.H.S
$(8)^2-(3)^2=4xy$
$64-9=4xy$
$55=4xy$
OR $4xy=55$
Divide by 4
$xy=\dfrac{55}{4}=13.75$
$\boxed{xy=13.75}$ Ans.

ii) $x+y=10,\ x-y=7$
$(x+y)^2+(x-y)^2=2(x^2+y^2)$
Putting the values in L.H.S
$(10)^2+(7)^2=2(x^2+y^2)$
$100+49=2(x^2+y^2)$
$149=2(x^2+y^2)$
or $2(x^2+y^2)=149$
$x^2+y^2=\dfrac{149}{2}$
Now use formula $(x+y)^2-(x-y)^2=4xy$
Putting values in L.H.S
$(10)^2-(7)^2=4xy$
$100-49=4xy$
$51=4xy$
$4xy=51\Rightarrow\boxed{xy=\dfrac{51}{4}}$ Ans.

iii) $x+y=11,\ x-y=5$
Use formula $(x+y)^2+(x-y)=2(x^2+y^2)$

Putting values in L.H.S

$(11)^2 + (5)^2 = 2(x^2 + y^2)$

$121 + 25 = 2(x^2 + y^2)$

$146 = 2(x^2 + y^2)$

$2(x^2 + y^2) = 146$

OR $x^2 + y^2 = \frac{146}{2} = 73$ Ans.

Now use formula $(x+y)^2 - (x-y)^2 = 4xy$

Putting values in L.H.S

$(11)^2 - (5)^2 = 4xy$

$121 - 25 = 4xy$

$96 = 4xy$ or $4xy = 96$

(Divide by 4)

OR $xy = \frac{\cancel{96}^{24}}{\cancel{4}} = 24$

$\Rightarrow$ $\boxed{xy = 24}$ Ans.

iv) $x + y = 7$, $x - y = 4$

As $(x+y)^2 + (x-y)^2 = 2(x^2 + y^2)$

$2(x^2 + y^2) = (x+y)^2 + (x-y)^2$

Putting the values in R.H.S

$= (7)^2 + (4)^2$

$= 49 + 16 = 65$

$2(x^2 + y)^2 = 65$

$x^2 + y^2 = \frac{65}{2}$ Ans.

Now $4xy = (x+y)^2 - (x-y)^2$

Putting the values in R.H.S

$= (7)^2 - (4)^2$

$= 49 - 16 = 33$

$4xy = 33 \Rightarrow \boxed{xy = \frac{33}{4}}$ Ans.

Q2: i) Find the values of $a^2 + b^2$ and ab, when $a + b = 7$, $a - b = 3$.

ii) Find the values of $a^2 + b^2$ and ab, when $a + b = 9$, $a - b = 1$.

Solution:

i) $(a+b)^2 + (a-b)^2 = 2(a^2 + b^2)$

Putting the values in L.H.S

$(7)^2 + (3)^2 = 2(a^2 + b^2)$

$49 + 9 = 2(a^2 + b^2)$

$2(a^2 + b^2) = 49 + 9$

$2(a^2 + b^2) = 58$

(Divide by 2)

$\frac{\cancel{2}(a^2 + b^2)}{\cancel{2}} = \frac{58}{2}$

$\Rightarrow a^2 + b^2 = \frac{\cancel{58}^{29}}{\cancel{2}}$

$\Rightarrow \boxed{a^2 + b^2 = 29}$ Ans.

Now $(a+b)^2 - (a-b)^2 = 4ab$

Putting the values in L.H.S

$(7)^2 - (3)^2 = 4ab$

$49 - 9 = 4ab$

$40 = 4ab$

$4ab = 40$

Dividing by 4

$ab = \frac{\cancel{40}^{10}}{\cancel{4}} = 10$

$\Rightarrow \boxed{ab = 10}$

ii) $(a+b)^2 + (a-b)^2 = 2(a^2 + b^2)$

Putting the values in L.H.S

$(9)^2 + (1)^2 = 2(a^2 + b^2)$

$81 + 1 = 2(a^2 + b^2)$

$82 = 2(a^2 + b^2)$ or $2(a^2 + b^2) = 82$

$\frac{\cancel{2}(a^2 + b^2)}{\cancel{2}} = \frac{\cancel{82}^{41}}{\cancel{2}}$

$\Rightarrow \boxed{a^2 + b^2 = 41}$ Ans.

Now $(a+b)^2 - (a-b)^2 = 4ab$

Putting the values in L.H.S

$(9)^2 - (1)^2 = 4ab$

$81 - 1 = 4ab$

$80 = 4ab$

$4ab = 80$

$ab = \frac{\cancel{80}^{20}}{\cancel{4}} \Rightarrow \boxed{ab = 20}$ Ans.

Q3: i) If $a+b=10$, $a-b=6$, then find the value of a^2+b^2.

ii) If $a+b=5$, $a-b=\sqrt{17}$ then find the value of ab.

Solution:

i) Formula $2(a^2+b^2)=(a+b)^2+(a-b)^2$

Putting values in R.H.S, we get

$2(a^2+b^2)=(10)^2+(6)^2$

$2(a^2+b^2)=100+36$ (Divide by 2)

$2(a^2-b^2)=136$

$\frac{\cancel{2}(a^2+b^2)}{\cancel{2}}=\frac{136}{2}$

$a^2+b^2=\frac{\cancel{136}^{68}}{\cancel{2}}=68$

$\Rightarrow$ $\boxed{a^2+b^2=68}$ Ans.

ii) Formula: $4ab=(a+b)^2-(a-b)^2$

Putting values in R.H.S

$4ab=(5)^2-(\sqrt{17})^2$

$4ab=25-17=8$

$4ab=8$ (Divide by 4)

$\frac{\cancel{4}ab}{\cancel{4}}=\frac{\cancel{8}^2}{\cancel{4}} \Rightarrow \boxed{ab=2}$ Ans.

Q4: Find the value of $4xy$ when $x+y=17$ and $x-y=5$.

Solution:

Formula: $4xy=(x+y)^2-(x-y)^2$

Putting the values in R.H.S

$4xy=(17)^2-(5)^2$

$\Rightarrow 4xy=289-25$

$\Rightarrow \boxed{4xy=264}$ Ans.

Q5: If $x+y=11$ and $x-y=3$, find $8xy(x^2+y^2)$.

Solution:

Formula; $2(x^2+y^2)=(x+y)^2+(x-y)^2$

Putting values in R.H.S, we get

$2(x^2+y^2)=(11)^2+(3)^2$

$2(x^2+y^2)=121+9=130 \rightarrow (1)$

Now using the formula

$4xy=(x+y)^2-(x-y)^2$

Putting values in R.H.S

$4xy=(11)^2-(3)^2$

$4xy=121-9=112 \rightarrow (2)$

Multiply (1) and (2), we get

$2(x^2+y^2)\times 4xy=130\times 112$

$8(x^2+y^2)xy=14560$

OR $\boxed{8xy(x^2+y^2)=14560}$ Ans.

Q6: If $u+v=7$ and $uv=12$. Find u – v.

Solution:

Formula: $4uv=(u+v)^2-(u-v)^2$

OR $(u-v)^2=(u+v)^2-4uv \rightarrow (1)$

Putting the values in R.H.S of equation (1)

$(u-v)^2=(7)^2-4(12)$

$(u-v)^2=49-48=1$

Taking square root on both sides

$\sqrt{(u-v)^2}=\sqrt{1}$

$\Rightarrow \boxed{u-v=\pm 1}$ Ans.

Derivation of the Formula $(a+b+c)^2=a^2+b^2+c^2+2ab+2bc+2ca$:

Proof: Let $b+c=x$, then taking L.H.S of above formula

$(a+b+c)^2=(a+x)^2=a^2+2ax+x^2 \rightarrow (1)$

Puting the values of x in equation (1)

$(a+b+c)^2=a^2+2a(b+c)+(b+c)^2$

$=a^2+2ab+2ac+b^2+c^2+2bc$

$=a^2+b^2+c^2+2ab+2bc+2ca$

EXAMPLE

Find the value of $a^2+b^2+c^2$, when $a+b+c=9$, $ab+bc+ca=20$.

Solution: Given

$a+b+c=9 \rightarrow (1)$

$ab+bc+ca=20 \rightarrow (2)$

Squaring (1) of both sides

$(a+b+c)^2=(9)^2$

$\Rightarrow a^2+b^2+c^2+2ab+2bc+2ca=81$

$\Rightarrow$

$a^2+b^2+c^2+2(ab+bc+ca)=81 \rightarrow (3)$

Put equation (2) in equation (3)

$\Rightarrow a^2+b^2+c^2+2(20)=81$

$\Rightarrow a^2+b^2+c^2+40=81$

$\Rightarrow a^2+b^2+c^2=81-40$

$\Rightarrow a^2+b^2+c^2=41$

EXAMPLE 14

Find the value of $a+b+c$, when a^2+b^2 $+c^2=29$, and $ab+bc+ca=26$.

Solution:

$(a+b+c)^2=a^2+b^2+c^2$

$+2(ab+bc+ca) \longrightarrow (1)$

Given that $a^2+b^2+c^2=29$ and $ab+bc+ca=26$

Putting these values in equation (1), we get

$(a+b+c)^2=29+2(26)$

$=29+52=-81$

$\because \quad a+b+c=\sqrt{81}=9$

EXAMPLE 15

Find the value of $ab+bc+ca$, where $a+b+c=14$ and $a^2+b^2+c^2=78$.

Solution:

$(a+b+c)^2=12^2 \Rightarrow$

$a^2+b^2+c^2+2(ab+bc+ca)=144 \rightarrow (1)$

Putting the value of $a^2+b^2+c^2=78$ in equation (1), we get

$78+2(ab+bc+ca)=144$

$2(ab+bc+ca)=144-78=66$

$\because ab+bc+ca=\dfrac{66}{2}=33$

EXERCISE 4.4

Q1: Find the values of $a^2+b^2+c^2$,

i) $a+b+c=5$ and $ab+bc+ca=-4$

ii) $a+b+c=5$ and $ab+bc+ca=-2$

Solution:

i) $a+b+c=5$ and $ab+bc+ca=-4$

Given $a+b=c=5$

Taking square on both sides

$(a+b+c)^2=(5)^2$

$a^2+b^2+c^2+2ab+2bc+2ca=25$

$a^2+b^2+c^2+2(ab+bc+ca)=25$

Putting the value of $ab+bc+ca=-4$

$a^2+b^2+c^2+2(-4)=25$

$a^2+b^2+c^2-8=25$

$\Rightarrow a^2+b^2+c^2=25+8$

$\Rightarrow \boxed{a^2+b^2+c^2=33}$ Ans.

ii) $a+b+c=5$ and $ab+bc+ca=-2$

Given $a+b+c=5$

Taking square on both sides, we get

$\Rightarrow (a+b+c)^2=(5)^2$

$\Rightarrow a^2+b^2+c^2+2ab+2bc+2ca=25$

OR $a^2+b^2+c^2+2(ab+bc+ca)=25$

Putting the value of $ab+bc+ca=-2$

OR $a^2+b^2+c^2+2(-2)=25$

$a^2+b^2+c^2-4=25$

$a^2+b^2+c^2=25+4=29$

Hence $\boxed{a^2+b^2+c^2=29}$ Ans.

Q2: Find the value of $a+b+c$, when:

i) $a^2+b^2+c^2=38$ & $ab+bc+ca=-1$

ii) $a^2+b^2+c^2=10$ & $ab+bc+ca=11$

Solution:

i) $a^2+b^2+c^2=38$ & $ab+bc+ca=-1$

We know from formula

$(a+b+c)^2=a^2+b^2+c^2+2(ab+bc+ca) \rightarrow (1)$

Putting the values in R.H.S of equation (1)

$(a+b+c)^2=38+2(-1)$

$=38-2=36$

$\sqrt{(a+b+c)^2}=\sqrt{36}$

Taking square root on both sides

$\Rightarrow (a+b+c)=\sqrt{6\times 6}$

$\Rightarrow \boxed{(a+b+c)=6}$ Ans.

ii) $a^2+b^2+c^2=10$ & $ab+bc+ca=11$

We know from formula

$(a+b+c)^2=a^2+b^2+c^2+2ab+2bc+2ca$

$(a+b+c)^2=a^2+b^2+c^2+2(ab+bc+ca) \rightarrow (1)$

Putting the values in R.H.S of equation (1)

$(a+b+c)^2=10+2(11)$

$=10+22=32$

Taking square root

$\sqrt{(a+b+c)^2}=\sqrt{32}$

$a+b+c=\sqrt{16\times 2}$

$\Rightarrow \boxed{(a+b+c)=4\sqrt{2}}$ Ans.

Q3: Find the value of $ab+bc+ca$. when:

i) $a^2+b^2+c^2=56$ & $a+b+c=12$

ii) $a^2+b^2+c^2=12$ & $a+b+c=5$

Solution:

i) $a^2+b^2+c^2=56$ & $a+b+c=12$

Given $a+b+c=12$

Squaring both sides, we get

$(a+b+c)^2=(12)^2$

$a^2+b^2+c^2+2ab+2bc+2ca=144$

$a^2+b^2+c^2+2(ab+bc+ca)=144$

Putting values of $a^2+b^2+c^2=56$

$56+2(ab+bc+ca)=144$

$2(ab+bc+ca)=144-56=88$

Divide by 2 both sides

$\frac{\cancel{2}(ab+bc+ca)}{\cancel{2}}=\frac{88}{2}$

$ab+bc+ca=\frac{\cancel{88}^{44}}{\cancel{2}}=44$

$\Rightarrow \boxed{ab+bc+ca=44}$ Ans.

ii) $a^2+b^2+c^2=12$ & $a+b+c=5$

Given that $a+b+c=5$

$(a+b+c)^2=(5)^2$

Squaring both sides, we get

$a^2+b^2+c^2+2ab+2bc+2ca=25$

$a^2+b^2+c^2+2(ab+bc+ca)=25$

Putting the value $a^2+b^2+c^2=12$

$12+2(ab+bc+ca)=25$

$2(ab+bc+ca)=25-12=13$

Divide by 2

$\frac{\cancel{2}(ab+bc+ca)}{\cancel{2}}=\frac{13}{2}$

$\Rightarrow \boxed{ab+bc+ca=6.5}$ Ans.

Q4: Prove that $x^2+y^2+z^2-xy-yz-zx$ $=\left(\frac{x-y}{\sqrt{2}}\right)^2+\left(\frac{y-z}{\sqrt{2}}\right)^2+\left(\frac{z-x}{\sqrt{2}}\right)^2$.

Solution:

We have to prove that

$x^2+y^2+z^2-xy-yz-zx$

$=\left(\frac{x-y}{\sqrt{2}}\right)^2+\left(\frac{y-z}{\sqrt{2}}\right)^2+\left(\frac{z-x}{\sqrt{2}}\right)^2$

$L.H.S=x^2+y^2+z^2-xy-yz-zx$

Divide and multiply by 2

$=\frac{2}{2}\left[x^2+y^2+z^2-xy-yz-zx\right]$

$=\frac{1}{2}\left[2x^2+2y^2+2z^2-2xy-2yz-2zx\right]$

$=\frac{1}{2}\left[x^2+x^2+y^2+y^2+z^2+z^2-2xy-2yz-2zx\right]$

$=\frac{1}{2}\left[x^2+y^2-2xy+y^2+z^2-2yz+z^2\right.$

$\left.+x^2-2zx\right]$

$=\frac{(x-y)^2}{2}+\frac{(y-z)^2}{2}+\frac{(z-x)^2}{2}$

$=\left(\frac{x-y}{\sqrt{2}}\right)^2+\left(\frac{y-z}{\sqrt{2}}\right)^2\left(\frac{z-x}{\sqrt{2}}\right)^2$

Hence $L.H.S=R.H.S$

Q5: Write $2\left[x^2+y^2+z^2-xy-yz-zx\right]$ as the sum of three squares.

Solution:

Given $2\left[x^2+y^2+z^2-xy-yz-zx\right]$

$=2x^2+2y^2+2z^2-2xy-2yz-2zx$

$= x^2 + x^2 + y^2 + y^2 + z^2 + z^2 - 2xy - 2yz - 2zx$

$= x^2 + y^2 - 2xy + y^2 + z^2 - 2yz + z^2 + x^2 - 2zx$

$= (x-y)^2 + (y-z)^2 + (z-x)^2$

Which is the sum of the three squares.

Q6: Find the value of $a^2 + b^2 + c^2 - ab - bc - ca$ when $a - b = 2, b - c = 3, c - a = 4$.

Solution:

We know from a result that

$2\left[a^2 + b^2 + c^2 - ab - bc - ca\right]$

$= (a-b)^2 + (b-c)^2 + (c-a)^2$

Divide both sides by 2, we get

$a^2 + b^2 + c^2 - ab - bc - ca$

$= \frac{1}{2}\left[(a-b)^2 + (b-c)^2 + (c-a)^2\right]$

Putting the values

$a - b = 2, b - c = 3, c - a = 4$

$= \frac{1}{2}\left[(2)^2 + (3)^2 + (4)^2\right]$

$= \frac{1}{2}[4 + 9 + 16]$

$= \frac{1}{2}[29] = \frac{29}{2} = 14.5$ Ans.

Hence $\boxed{a^2 + b^2 + c^2 - ab - bc - ca = 14.5}$

Derivation of Formula:

i) $(a+b)^3 = a^3 + 3ab(a+b) + b^3$

ii) $(a-b)^3 = a^3 - 3ab(a-b) - b^3$

Proof (i) $(a+b)^3 = a^3 + 3ab(a+b) + b^3$

$(a+b)^3 = (a+b)(a+b)^2$

$= (a+b)(a^2 + 2ab + b^2)$

$= a.a^2 + 2a.a.b + ab^2 + ba^2 + 2a.b.b + b.b^2$

$= a^3 + 2a^2b + ab^2 + a^2b + 2ab^2 + b^3$

$= a^3 + 3a^2b + 3ab^2 + b^3$

$\Rightarrow (a+b)^3 = a^3 + 3ab(a+b) + b^3$

Derivation of Formula:

Proof (ii) $(a-b)^3 = a^3 - 3ab(a-b) - b^3$

$(a-b)^3 = (a-b)(a-b)^2$

$= (a-b)(a^2 - 2ab + b^2)$

$= a.a^2 - 2a.a.b + ab^2 - ba^2 + 2a.b.b - b.b^2$

$= a^3 - 2a^2b + ab^2 - a^2b + 2ab^2 - b^3$

$= a^3 - 3a^2b + 3ab^2 - b^3$

$\Rightarrow (a-b)^3 = a^3 - 3ab(a-b) - b^3$

EXAMPLE

Find the value of $a^3 + b^3$, when $a + b = 5$ and $ab = 10$.

Solution:

$(a+b)^3 = a^3 + 3ab(a+b) + b^3$

$5^3 = a^3 + b^3 + 3 \times 10(5)$

Putting the values of $a+b$ and ab

$125 = a^3 + b^3 + 50$

$\therefore a^3 + b^3 = 125 - 50 = 75$

Derivation of Formula:

Proof (i) $\left(x + \frac{1}{x}\right)^3 = x^3 + \frac{1}{x^3} + 3\left(x + \frac{1}{x}\right)$

Since we know that

$(a+b)^3 = a^3 + 3ab(a+b) + b^3 \rightarrow (1)$

Put $a = x, b = \frac{1}{x}$ in equation (1)

$\left(x + \frac{1}{x}\right)^3 = x^3 + 3(x)\left(x + \frac{1}{x}\right) + \frac{1}{x^3}$

$\Rightarrow \left(x + \frac{1}{x}\right)^3 = x^3 + 3\left(x + \frac{1}{x}\right) + \frac{1}{x^3}$

Derivation of Formula:

Proof (ii) $\left(x - \frac{1}{x}\right)^3 = x^3 - \frac{1}{x^3} + 3\left(x - \frac{1}{x}\right)$

Since we know that

$(a-b)^3 = a^3 - 3ab(a-b) - b^3 \rightarrow (1)$

Put $a = x, b = \frac{1}{x}$ in equation (1)

$\left(x - \frac{1}{x}\right)^3 = x^3 - 3(x)\left(x - \frac{1}{x}\right) - \frac{1}{x^3}$

$\Rightarrow \left(x - \frac{1}{x}\right)^3 = x^3 - 3\left(x - \frac{1}{x}\right) - \frac{1}{x^3}$

EXAMPLE

Find the value of $x^3+\frac{1}{x^3}$, when $x+\frac{1}{x}=5$.

Solution:

$\left(x+\frac{1}{x}\right)^3=x^3+\frac{1}{x^3}+3\left(x+\frac{1}{x}\right)\rightarrow(1)$

Putting the value of $x+\frac{1}{x}=5$ in equation (1), we get

$(5)^3=x^3+\frac{1}{x^3}+3.5$

or $x^3+\frac{1}{x^3}=125-15=110$

EXERCISE 4.5

Q1: Find the value of a^3+b^3, when:

i) $a+b=4$ **and** $ab=5$
ii) $a+b=3$ **and** $ab=20$
iii) $a+b=4$ **and** $ab=2$

Solution:

i) $a+b=4$ **and** $ab=5$

Given that $a+b=4$

Taking cube on both sides

$(a+b)^3=(4)^3$

$a^3+b^3+3ab(a+b)=64$

Putting the values of $a+b=4$ and $ab=5$

$a^3+b^3+3(5)(4)=64$

$a^3+b^3+60=64$

$a^3+b^3=64-60$

$\Rightarrow \boxed{a^3+b^3=4}$ Ans.

ii) $a+b=3$ **and** $ab=20$

Given $a+b=3$

Taking cube on both sides

$(a+b)^3=(3)^3$

$a^3+b^3+3ab(a+b)=27$

Putting the values of $a+b=3$ and $ab=20$

$a^3+b^3+3(20)(3)=27$

$a^3+b^3+180=27$

$a^3+b^3=27-180$

$\Rightarrow \boxed{a^3+b^3=-153}$ Ans.

iii) $a+b=4$ **and** $ab=2$

Given $a+b=4$

Taking cube on both sides

$(a+b)^3=(4)^3$

$a^3+b^3+3ab(a+b)=64$

Putting the values of $ab=2$ and $a+b=4$

$a^3+b^3+3(2)(4)=64$

$a^3+b^3+24=64$

$a^3+b^3=64-24=40$

$\Rightarrow \boxed{a^3+b^3=40}$ Ans.

Q2: Find the value of a^3-b^3 when:

i) $a-b=5$ **and** $ab=7$
ii) $a-b=2$ **and** $ab=15$
iii) $a-b=7$ **and** $ab=6$.

Solution:

i) $a-b=5$ **and** $ab=7$

Given $a-b=5$

Taking cube on both sides

$(a-b)^3=(5)^3$

$\Rightarrow a^3-b^3-3ab(a-b)=125$

$\Rightarrow a^3-b^3-3(7)(5)=125$

$\Rightarrow a^3-b^3-105=125$

$\Rightarrow a^3-b^3=125+105=230$

$\therefore \boxed{a^3-b^3=230}$ Ans.

ii) $a-b=2$ **and** $ab=15$

Given $a-b=2$

Taking cube on both sides

$(a-b)^3=(2)^3$

$\Rightarrow a^3-b^3-3ab(a-b)=8$

Putting the values

$\Rightarrow a^3-b^3-3(15)(2)=8$

$\Rightarrow a^3-b^3-90=8$

$\Rightarrow a^3-b^3=8+90=98$

$\therefore \boxed{a^3-b^3=98}$ Ans.

iii) $a - b = 7$ **and** $ab = 6$

Given $a - b = 7$

Taking cube on both sides

$(a-b)^3 = (7)^3$

$\Rightarrow a^3 - b^3 - 3ab(a-b) = 343$

Putting the values

$\Rightarrow a^3 - b^3 - 3(6)(7) = 343$

$\Rightarrow a^3 - b^3 - 126 = 343$

$\Rightarrow a^3 - b^3 = 343 + 126$

$\Rightarrow a^3 - b^3 = 469$

$\therefore \boxed{a^3 - b^3 = 469}$ Ans.

Q3: Find the value of $x^3 + \frac{1}{x^3}$ when:

i) $x + \frac{1}{x} = \frac{5}{2}$ ii) $x + \frac{1}{x} = 2$

Solution:

i) $x + \frac{1}{x} = \frac{5}{2}$

Given $x + \frac{1}{x} = \frac{5}{2}$

Taking cube on both sides

$\left(x + \frac{1}{x}\right)^3 = \left(\frac{5}{2}\right)^3$

$\Rightarrow x^3 + \frac{1}{x^3} + 3(\cancel{x})\left(\frac{1}{\cancel{x}}\right)\left(x + \frac{1}{x}\right) = \frac{125}{8}$

Put value of $x + \frac{1}{x} = \frac{5}{2}$

$\Rightarrow x^3 + \frac{1}{x^3} + 3\left(\frac{5}{2}\right) = \frac{125}{8}$

$\Rightarrow x^3 + \frac{1}{x^3} + \frac{15}{2} = \frac{125}{8}$

$\Rightarrow x^3 + \frac{1}{x^3} = \frac{125}{8} - \frac{15}{2}$

$x^3 + \frac{1}{x^3} = \frac{125 - 60}{8} = \frac{65}{8}$

$\therefore \boxed{x^3 + \frac{1}{x^3} = \frac{65}{8}}$

ii) $x + \frac{1}{x} = 2$

Given $x + \frac{1}{x} = 2$

Taking cube on both sides

$\left(x + \frac{1}{x}\right)^3 = (2)^3$

$\Rightarrow x^3 + \frac{1}{x^3} + 3\left(x + \frac{1}{x}\right) = 8$

Put value of $x + \frac{1}{x} = 2$

$\Rightarrow x^3 + \frac{1}{x^3} + 3(2) = 8$

$\Rightarrow x^3 + \frac{1}{x^3} + 6 = 8$

$\Rightarrow x^3 + \frac{1}{x^3} = 8 - 6 = 2$

$\therefore \boxed{x^3 + \frac{1}{x^3} = 2}$ Ans.

Q4: Find the value of $x^3 - \frac{1}{x^3}$, when:

i) $x - \frac{1}{x} = \frac{3}{2}$ ii) $x - \frac{1}{x} = \frac{7}{3}$

iii) $x - \frac{1}{x} = \frac{15}{4}$

Solution:

i) $x - \frac{1}{x} = \frac{3}{2}$

Given $x - \frac{1}{x} = \frac{3}{2}$

Taking cube on both sides

$\left(x - \frac{1}{x}\right)^3 = \left(\frac{3}{2}\right)^3$

$x^3 - \frac{1}{x^3} - 3\left(x - \frac{1}{x}\right) = \frac{27}{8}$

Put the value $x - \frac{1}{x} = \frac{3}{2}$

$x^3 - \frac{1}{x^3} - 3\left(\frac{3}{2}\right) = \frac{27}{8}$

$$x^3-\frac{1}{x^3}-\frac{9}{2}=\frac{27}{8}$$

$$\Rightarrow x^3-\frac{1}{x^3}=\frac{27}{8}+\frac{9}{2}$$

$$x^3-\frac{1}{x^3}=\frac{27+36}{8}=\frac{63}{8}$$

$$\therefore \boxed{x^3-\frac{1}{x^3}=\frac{63}{8}} \quad \text{Ans.}$$

ii) $x-\frac{1}{x}=\frac{7}{3}$

Given $x-\frac{1}{x}=\frac{7}{3}$

Take cube on both sides

$$\left(x-\frac{1}{x}\right)^3=\left(\frac{7}{3}\right)^3$$

$$\Rightarrow x^3-\frac{1}{x^3}-3\left(x-\frac{1}{x}\right)=\frac{343}{27}$$

Put value $x-\frac{1}{x}=\frac{7}{3}$

$$x^3-\frac{1}{x^3}-\cancel{3}\left(\frac{7}{\cancel{3}}\right)=\frac{343}{27}$$

$$\Rightarrow x^3-\frac{1}{x^3}-7=\frac{343}{27}$$

$$\Rightarrow x^3-\frac{1}{x^3}=\frac{343}{27}+7$$

$$x^3-\frac{1}{x^3}=\frac{343+189}{27}=\frac{532}{27}$$

$$\therefore \boxed{x^3-\frac{1}{x^3}=\frac{532}{27}} \quad \text{Ans.}$$

iii) $x-\frac{1}{x}=\frac{15}{4}$

Given $x-\frac{1}{x}=\frac{15}{4}$

Taking cube on both sides

$$\left(x-\frac{1}{x}\right)^3=\left(\frac{15}{4}\right)^3$$

$$\Rightarrow x^3-\frac{1}{x^3}-3\left(x-\frac{1}{x}\right)=\frac{3375}{64}$$

Putting the values

$$x-\frac{1}{x}=\frac{15}{4}$$

$$\Rightarrow x^3-\frac{1}{x^3}-3\left(\frac{15}{4}\right)=\frac{3375}{64}$$

$$\Rightarrow x^3-\frac{1}{x^3}-\frac{45}{4}=\frac{3375}{64}$$

$$\Rightarrow x^3-\frac{1}{x^3}=\frac{3375}{64}+\frac{45}{4}$$

$$\Rightarrow x^3-\frac{1}{x^3}=\frac{3375+720}{64}=\frac{4095}{64}$$

$$\therefore \boxed{x^3-\frac{1}{x^3}=\frac{4095}{64}} \quad \text{Ans.}$$

Q5: If $3a+\frac{1}{a}=4$, find $27a^3+\frac{1}{a^3}$.

Solution:

Given that $3a+\frac{1}{a}=4$

Taking cube on both sides

$$\left(3a+\frac{1}{a}\right)^3=(4)^3$$

$$\Rightarrow (3a)^3+\left(\frac{1}{a}\right)^3+3(3\cancel{a})\left(\frac{1}{\cancel{a}}\right)\left(3a+\frac{1}{a}\right)$$

Putting the values

$$=27a^3+\frac{1}{a^3}+9(4)=64$$

$$\Rightarrow 27a^3+\frac{1}{a^3}+36=64$$

$$\Rightarrow 27a^3+\frac{1}{a^3}=64-36=28$$

$$\therefore \boxed{27a^3+\frac{1}{a^3}=28} \quad \text{Ans.}$$

Q6: If $x-\frac{1}{2x}=6$, find $x^3-\frac{1}{8x^3}$.

Solution:

Given that $x-\frac{1}{2x}=6$

Taking cube of both sides

$\left(x-\dfrac{1}{2x}\right)^3=(6)^3$

$\Rightarrow (x)^3-\left(\dfrac{1}{2x}\right)^3-3(\cancel{x})\left(\dfrac{1}{2\cancel{x}}\right)\left(x-\dfrac{1}{2x}\right)=216$

$x^3-\dfrac{1}{8x^3}-\dfrac{3}{2}\left(x-\dfrac{1}{2x}\right)=216$

Putting the values

$\Rightarrow x^3-\dfrac{1}{8x^3}-\dfrac{3}{\cancel{2}}(\cancel{6}^3)=216$

$\Rightarrow x^3-\dfrac{1}{8x^3}-9=216$

$\Rightarrow x^3-\dfrac{1}{8x^3}=216+9=225$

$\therefore \boxed{x^3-\dfrac{1}{8x^3}=225}$ Ans.

Q7: If $a+b=6$, show that $a^3+b^3+18ab=216$

Solution:

Given $a+b=6$

Taking cube on both sides, we get

$(a+b)^3=(6)^3$

$\Rightarrow (a+b)^3=(6)^3$

$\Rightarrow a^3+b^3+3ab(a+b)=216$

Put value of $a+b=6$

$\Rightarrow a^3+b^3+3ab(6)=216$

$\Rightarrow a^3+b^3+18ab=216$

$\therefore \boxed{a^3+b^3+18ab=216}$

Q8: If $u-v=3$, then prove that $u^3-b^3-9uv=27$.

Solution:

Given $u-v=3$

Taking cube on both sides

$(u-v)^3=(3)^3$

$\Rightarrow u^3-v^3-3uv(u-v)=27$

Put $u-v=3$

$\Rightarrow u^3-v^3-3uv(3)=27$

$\Rightarrow u^3-v^3-9uv=27$ proved.

Q9: If $a+\dfrac{1}{a}=2$, find the values of $a^2+\dfrac{1}{a^2}, a^4+\dfrac{1}{a^4}, a^3+\dfrac{1}{a^3}$.

Solution:

Given $a+\dfrac{1}{a}=2$

Squaring both sides, we get

$\left(a+\dfrac{1}{a}\right)^2=(2)^2$

$\Rightarrow a^2+\dfrac{1}{a^2}+2(\cancel{a})\left(\dfrac{1}{\cancel{a}}\right)=4$

$\Rightarrow a^2+\dfrac{1}{a^2}+2=4 \Rightarrow a^2+\dfrac{1}{a^2}=4-2=2$

$\therefore \boxed{a^2+\dfrac{1}{a^2}=2}$ Ans.

Taking square again both sides

$\left(a^2+\dfrac{1}{a^2}\right)^2=(2)^2$

$\Rightarrow (a^2)^2+\left(\dfrac{1}{a^2}\right)^2+2(\cancel{a^2})\left(\dfrac{1}{\cancel{a^2}}\right)=4$

$\Rightarrow a^4+\dfrac{1}{a^4}+2=4$

$\Rightarrow a^4+\dfrac{1}{a^4}=4-2=2$

$\therefore \boxed{a^4+\dfrac{1}{a^4}=2}$ Ans.

Now $a+\dfrac{1}{a}=2$.

Taking cube on both sides

$\left(a+\dfrac{1}{a^2}\right)^3=(2)^3$

$\Rightarrow a^3+\dfrac{1}{a^3}+3\left(a+\dfrac{1}{a}\right)=8$

Put $a+\dfrac{1}{a}=2$ we get

$\Rightarrow a^3+\dfrac{1}{a^3}+3(2)=8$

$\Rightarrow a^3 + \frac{1}{a^3} + 6 = 8$

$\Rightarrow a^3 + \frac{1}{a^3} = 8 - 2 = 2$

$\therefore \boxed{a^3 + \frac{1}{a^3} = 2}$

Hence

$a^2 + \frac{1}{a^2} = a^4 + \frac{1}{a^4} = a^3 + \frac{1}{a^3} = 2$ Ans.

Derivation of the Formula:

$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

Take R.H.S

$(a+b)(a^2 - ab + b)^2$

$= a.a^2 - a.ab + ab^2 + ba^2 - ab.b + b.b^2$

$= a^3 - a^2b + ab^2 + a^2b - ab^2 + b^3$

$= a^3 + b^3 =$ L.H.S

EXAMPLE 19

Find the value of $a^3 + b^3$, when $a + b = 4$, $ab = 3$.

Solution:

We know that

$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

$= (a+b)\{(a+b)^2 - 3ab\} \rightarrow (1)$

Putting $a+b=4$, $ab=3$ in equation (1), we get

$a^3 + b^3 = (4)(4^2 - 3.3)$

$= 4(16-9) = 4 \times 7 = 28$

Derivation of the Formula:

$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

Take R.H.S

$(a-b)(a^2 + ab + b^2)$

$= a.a^2 + a.ab + ab^2 - ba^2 - ab.b - b.b^2$

$= a^3 + a^2b + ab^2 - a^2b - ab^2 - b^3$

$= a^3 - b^3 =$ L.H.S

EXAMPLE 20

Find the value of $a^3 - b^3$, when $a - b = 2$, $ab = 10$.

Solution:

We know that

$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

$= (a-b)(a^2 + b^2 - 2ab + 2ab + ab)$

$= (a-b)\{(a-b)^2 + 3ab\} \rightarrow (1)$

Putting $a-b=2$ and $ab=10$ in equation (1), we get

$a^3 - b^3 = (2)(2^2 + 3 \times 10)$

$= 2(4+30) = 2 \times 34 = 68$

To find Continued Product:

$\left(x + \frac{1}{x}\right)\left(x^2 + \frac{1}{x^2} - 1\right)$

As $a^3 + b^3 = (a+b)(a^2 - ab + b^2) \rightarrow (1)$

Let $a = x$, $b = \frac{1}{x}$ put in equation (1)

Then $x^3 + \frac{1}{x^3} = \left(x + \frac{1}{x}\right)\left(x^2 - x.\frac{1}{x} + \frac{1}{x^2}\right)$

$\Rightarrow x^3 + \frac{1}{x^3} = \left(x + \frac{1}{x}\right)\left(x^2 - 1 + \frac{1}{x^2}\right)$

Thus $\left(x + \frac{1}{x}\right)\left(x^2 + \frac{1}{x^2} - 1\right) = x^3 + \frac{1}{x^3}$

To find Continued Product:

$\left(x - \frac{1}{x}\right)\left(x^2 + \frac{1}{x^2} + 1\right)$

As $a^3 - b^3 = (a-b)(a^2 + ab + b^2) \rightarrow (1)$

Let $a = x$, $b = \frac{1}{x}$ put in equation (1)

Then $x^3 - \frac{1}{x^3} = \left(x - \frac{1}{x}\right)\left(x^2 + x.\frac{1}{x} + \frac{1}{x^2}\right)$

$\Rightarrow x^3 - \frac{1}{x^3} = \left(x - \frac{1}{x}\right)\left(x^2 + 1 + \frac{1}{x^2}\right)$

Thus $\left(x - \frac{1}{x}\right)\left(x^2 + \frac{1}{x^2} + 1\right) = x^3 - \frac{1}{x^3}$

To find the Continued Product of:

$(x+y)(x-y)(x^2 + xy + y^2)(x^2 - xy + y^2)$

We have

$(x+y)(x^2 - xy + y^2).(x-y)(x^2 + xy + y^2)$

$= (x+y)(x^2 - xy + y^2).(x-y)(x^2 + xy + y^2)$

$=(x^3+y^3).(x^3-y^3)$

Using the formulas

$=(x^3)^2-(y^3)^2$

$=x^6-y^6$

EXERCISE 4.6

Q1: Find the following product:

i) $(a-1)(a^2+a+1)$

ii) $(3-b)(9+3b+b^2)$

iii) $(8+b)(64-8b+b^2)$

iv) $(a+2)(a^2-2a+4)$

Solution:

i) $(a-1)(a^2+a+1)$

Given $(a-1)(a^2+a+1)$

$=a.a^2+a.a+a.1-1.a^2-1.a-1.1$

$=a^3+\cancel{a^2}+\cancel{a}-\cancel{a^2}-\cancel{a}-1=a^3-1$ Ans.

ii) $(3-b)(9+3b+b^2)$

Given $(3-b)(9+3b+b^2)$

$=3\times9+3\times3b+3\times b^2-b\times9-b\times3b-b\times b^2$

$=27+\cancel{9b}+\cancel{3b^2}-\cancel{9b}-\cancel{3b^2}-b^3$

$=27-b^3$ Ans.

iii) $(8+b)(64-8b+b^2)$

Given $(8+b)(64-8b+b^2)$

$8\times64-8\times8b+8\times b^2+b\times64-b\times8b+b\times b^2$

$=512-\cancel{64b}+\cancel{8b^2}+\cancel{64b}-\cancel{8b^2}+b^3$

$=512+b^3$ Ans.

iv) $(a+2)(a^2-2a+4)$

Given $(a+2)(a^2-2a+4)$

$a\times a^2-a\times2a+a\times4+2\times a^2-2\times2a+2\times4$

$=a^3-\cancel{2a^2}+\cancel{4a}+\cancel{2a^2}-\cancel{4a}+8$

$=a^3+8$ Ans.

Q2: Find the following product:

i) $\left(2p+\frac{1}{2p}\right)\left(4p^2+\frac{1}{4p^2}-1\right)$

ii) $\left(\frac{3}{2}p-\frac{2}{3p}\right)\left(\frac{9}{4}p^2+\frac{4}{9p^2}+1\right)$

iii) $\left(3p-\frac{1}{3p}\right)\left(9p^2+\frac{1}{9p^2}+1\right)$

iv) $\left(5p+\frac{1}{5p}\right)\left(25p^2+\frac{1}{25p^2}-1\right)$

Solution:

i) $\left(2p+\frac{1}{2p}\right)\left(4p^2+\frac{1}{4p^2}-1\right)$

$=2p\times4p^2+\cancel{2}\cancel{p}\times\frac{1}{{}_2\cancel{4}\cancel{p^2}}-2p\times1+\frac{1}{\cancel{2}\cancel{p}}\times{}^2\cancel{4}\cancel{p^2}+\frac{1}{2p}\times\frac{1}{4p^2}-\frac{1}{2p}\times1$

$=8p^3+\frac{1}{\cancel{2p}}-\cancel{2p}+\cancel{2p}+\frac{1}{8p^3}-\frac{1}{\cancel{2p}}$

$=8p^3+\frac{1}{8p^3}$ Ans.

ii) $\left(\frac{3}{2}p-\frac{2}{3p}\right)\left(\frac{9}{4}p^2+\frac{4}{9p^2}+1\right)$

$=\frac{3}{2}p\times\frac{9}{4}p^2+\frac{\cancel{3}}{\cancel{2}}\cancel{p}\times\frac{{}^2\cancel{4}}{{}_3\cancel{9}\cancel{p^2}}+\frac{3}{2}p\times1$

$-\frac{\cancel{2}}{\cancel{3}\cancel{p}}\times\frac{\cancel{9}^3}{\cancel{4}_2}\cancel{p^2}-\frac{3}{2p}\times\frac{4}{9p^2}-\frac{2}{3p}\times1$

$=\frac{27}{8}p^3+\frac{2}{\cancel{3p}}+\frac{3}{\cancel{2}}p-\frac{3}{\cancel{2}}p-\frac{8}{27p^3}-\frac{2}{\cancel{3p}}$

$=\frac{27}{8}p^3-\frac{8}{27p^3}$ Ans.

iii) $\left(3p-\frac{1}{3p}\right)\left(9p^2+\frac{1}{9p^2}+1\right)$

$=3p\times9p^2+3p\times\frac{1}{9p^2}+3p\times1-\frac{1}{3p}\times9p^2$

$-\frac{1}{3p}\times\frac{1}{9p^2}-\frac{1}{3p}\times1$

$=27p^3+\frac{1}{\cancel{3p}}+\cancel{3p}-\cancel{3p}-\frac{1}{27p^3}-\frac{1}{\cancel{3p}}$

$=27p^3-\frac{1}{27p^3}$ Ans.

iv) $\left(5p+\frac{1}{5p}\right)\left(25p^2+\frac{1}{25p^2}-1\right)$

$= 5p\times25p^2+5p\times\frac{1}{25p^2}-5p\times1+\frac{1}{5p}\times25p^2$

$+\frac{1}{5p}\times\frac{1}{25p^2}-\frac{1}{5p}\times1$

$=125p^3+\cancel{\frac{1}{5p}}-\cancel{5p}+\cancel{5p}+\frac{1}{125p^3}-\cancel{\frac{1}{5p}}$

$=125p^3+\frac{1}{125p^3}$ Ans.

Q3: Find the following continued product:

i) $(x^2-y^2)(x^2-xy+y^2)(x^2+xy+y^2)$

ii) $(x+y)(x-y)(x^2+y^2)(x^4+y^4)$

iii) $(2x-y)(2x+y)(4x^2-2xy+y^2)(4x^2-2xy+y^2)$

iv) $(x-2)(x+2)(x^2-2x+4)(x^2+2x+4)$

Solution:

i) $(x^2-y^2)(x^2-xy+y^2)(x^2+xy+y^2)$

Re-arranging the terms

$(x+y)(x-y)(x^2-xy+y^2)(x^2+xy+y^2)$

$=(x+y)(x^2-xy+y)(x-y)(x^2+xy+y^2)$

Using formulas

$=\left[(x)^3+(y)^3\right]\left[(x)^3-(y)^3\right]$

$=\left(x^3+y^3\right)\left(x^3-y^3\right)$ $\because (a+b)(a-b)$

$=\left(x^3\right)^2-\left(y^3\right)^2$ $=a^2-b^2$

$\Rightarrow x^6-y^6$ Ans.

ii) $(x+y)(x-y)(x^2+y^2)(x^4+y^4)$

Given $(x+y)(x-y)(x^2+y^2)(x^4+y^4)$

$=\left[(x)^2-(y)^2\right](x^2+y^2)(x^4+y^4)$

$=\left(x^2-y^2\right)\left(x^2+y^2\right)\left(x^4+y^4\right)$

$=\left[\left(x^2\right)^2-\left(y^2\right)^2\right]\left(x^4+y^4\right)$

$=\left(x^4-y^4\right)\left(x^4+y^4\right)$

$=\left(x^4\right)^2-\left(y^4\right)^2$

$\Rightarrow x^8-y^8$ Ans.

iii) $(2x-y)(2x+y)(4x^2-2xy+y^2)(4x^2-2xy+y^2)$

By re-arranging the factors

$=(2x-y)\left(4x^2+2xy+y^2\right)$

$(2x+y)\left(4x^2-2xy+y^2\right)$

$=\left[(2x)^3-(y)^3\right]\left[(2x)^3+(y)^3\right]$

$=\left(8x^3-y^3\right)\left(8x^3+y^3\right)$

$\Rightarrow\left(8x^3\right)^2-\left(y^3\right)^2$

$=64x^6-y^6$ Ans.

iv) $(x-2)(x+2)(x^2-2x+4)(x^2+2x+4)$

By re-arranging the factors

$(x-2)\left(x^2+2x+4\right)\left(x+2\right)\left(x^2-2x+4\right)$

$=\left[(x)^3-(2)^3\right]\left[(x)^3+(2)^3\right]$

$\Rightarrow(x^3-8)(x^3+8)$

As $(a-b)(a+b)=a^2-b^2$

$=\left(x^3\right)^2-(8)^2\Rightarrow x^6-64$ Ans.

Q4: Find the product with the help of formula $\left(\sqrt{x}-\sqrt{y}\right)\left(x+\sqrt{xy}+y\right)$.

Solution:

Given $\left(\sqrt{x}-\sqrt{y}\right)\left(x+\sqrt{xy}+y\right)$

Formula is

$(a-b)(a^2+ab+b^2)=a^3-b^3$

$=\left(\sqrt{x}-\sqrt{y}\right)\left[\left(\sqrt{x}\right)^2+\left(\sqrt{x}\right)\left(\sqrt{y}\right)+\left(\sqrt{y}\right)^2\right]$

$=\left(\sqrt{x}\right)^3-\left(\sqrt{y}\right)^3$

$=\sqrt{x.x.x}-\sqrt{y.y.y}$

$=\boxed{x\sqrt{x}-y\sqrt{y}}$ Ans.

Q5: Simplify with the help of formula $(x^p+y^q)(x^{2p}-xy^{pq}+y^{2q})$.

Solution:

Given $(x^p+y^q)(x^{2p}-(xy)^{pq}+y^{2q})$

Formula is

$(a+b)(a^2+ab+b^2)=a^3+b^3$

$=(x^p+y^q)\left[(x^p)^2-(x^p)(y^q)+(y^q)^2\right]$

$=(x^p+y^q)\left[x^{2p}-x^py^q+y^{2q}\right]$

$=(x^p)^3+(y^q)^3=x^{3p}+y^{3q}$ Ans.

Surds and Their Application:

Surds: A number of the form $\sqrt[n]{a}$ where a is a positive rational number is called a surd.

Note: A number will be a surd if:

i) It is an irrational number

ii) It is a root

iii) It is a root of rational number

For example $\sqrt{3}$ and $\sqrt{5+\sqrt{3}}$ are both irrational numbers. First number $\sqrt{3}$ is a root of a rational number 3. The 2nd number $\sqrt{5+\sqrt{3}}$ is a root of an irrational number $5+\sqrt{3}$. So the first number $\sqrt{3}$ is a surd. The number $\sqrt[3]{8}$ is not a surd. Similarly $\sqrt{-2},\sqrt{-3}$ are not surds because -2 and -3 are negative.

Order of a Surd:

In a surd $\sqrt[n]{a}$ here n is called the order of the surd. (OR) The surd index and a is called the radicand.

EXAMPLE

If $x=2+\sqrt{3}$, find the values of $x+\frac{1}{x}$ and $x^2+\frac{1}{x^2}$.

Solution:

Since $x=2+\sqrt{3}, \frac{1}{x}=\frac{1}{2+\sqrt{3}}$

$\frac{1}{x}=\frac{2-\sqrt{3}}{(2+\sqrt{3})(2-\sqrt{3})}$

Multiplying the denominator and numerator by $2-\sqrt{3}$ i.e. by the conjugate of $2+\sqrt{3}$

$\frac{1}{x}=\frac{2-\sqrt{3}}{4-3}=2-\sqrt{3}$

$\therefore x+\frac{1}{x}=(2+\sqrt{3})+(2-\sqrt{3})=4$

Now, $\left(x+\frac{1}{x}\right)^2=16 \Rightarrow x^2+\frac{1}{x^2}+2=16$

$\therefore x^2+\frac{1}{x^2}=14$

EXAMPLE

If $x=\sqrt{3}-\sqrt{2}$, find the values of $x-\frac{1}{x}$ and $x^2+\frac{1}{x^2}$.

Solution:

Since $x=\sqrt{3}-\sqrt{2}, \frac{1}{x}=\frac{1}{\sqrt{3}-\sqrt{2}}$

$\frac{1}{x}=\frac{\sqrt{3}+\sqrt{2}}{(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2})}$

$=\frac{\sqrt{3}+\sqrt{2}}{3-2}=\sqrt{3}+\sqrt{2}$

$\therefore x-\frac{1}{x}=(\sqrt{3}-\sqrt{2})-(\sqrt{3}+\sqrt{2})=-2\sqrt{2}$

Now, $\left(x-\frac{1}{x}\right)^2=\left(-2\sqrt{2}\right)^2$

$\Rightarrow x^2+\frac{1}{x^2}-2=4\times 2=8$

$\therefore x^2+\frac{1}{x^2}=10$

EXERCISE 4.7

Q1: State which of the following are surd quantities?

i) $\sqrt[3]{81}$	ii) $\sqrt{1+\sqrt{5}}$
iii) $\sqrt{\sqrt{5}}$	iv) $\sqrt[4]{32}$
v) π	vi) $\sqrt{1+\pi^2}$

Solution:

i) $\sqrt[3]{81}$

Ans. This expression is a surd.

ii) $\sqrt{1+\sqrt{5}}$

Ans. This expression is a surd.

iii) $\sqrt{\sqrt{5}}$

Ans. This expression is not a surd.

iv) $\sqrt[4]{32}$

Ans. This expression is not a surd.

v) π

Ans. This expression is not a surd.

vi) $\sqrt{1+\pi^2}$

Ans. This expression is also a surd.

Q2: Express the following as the simplest possible surds:

i) $\sqrt{12}$ ii) $\sqrt{48}$

iii) $\sqrt{240}$

Solution:

i) $\sqrt{12}$

Given $\sqrt{12} = \sqrt{2\times2\times3}$

$= 2\sqrt{3}$ Ans.

ii) $\sqrt{48}$

Given $\sqrt{48} = \sqrt{16\times3}$

$= \sqrt{4\times4\times3}$

$= 4\sqrt{3}$ Ans.

iii) $\sqrt{240}$

Given $\sqrt{240} = \sqrt{2\times2\times2\times2\times15}$

$= 2\times2\sqrt{15} = 4\sqrt{15}$ Ans.

Q3: Simplify the following surds:

i) $(2-\sqrt{3})(3+\sqrt{5})$

ii) $(\sqrt{3}-4)(\sqrt{2}+1)$

iii) $(\sqrt{2}+\sqrt{3})(\sqrt{5}+\sqrt{2})$

iv) $(3-2\sqrt{3})(3+2\sqrt{3})$

Solution:

i) $(2-\sqrt{3})(3+\sqrt{5})$

Given $(2-\sqrt{3})(3+\sqrt{5})$

$= 2\times3+2\sqrt{5}-3\sqrt{3}-\sqrt{3}\times\sqrt{5}$

$= 6+2\sqrt{5}-3\sqrt{3}-\sqrt{15}$ Ans.

ii) $(\sqrt{3}-4)(\sqrt{2}+1)$

Given $(\sqrt{3}-4)(\sqrt{2}+1)$

$= \sqrt{3}\times\sqrt{2}+\sqrt{3}\times1-4\sqrt{2}-4\times1$

$= \sqrt{6}+\sqrt{3}-4\sqrt{2}-4$ Ans.

iii) $(\sqrt{2}+\sqrt{3})(\sqrt{5}+\sqrt{2})$

Given $(\sqrt{2}+\sqrt{3})(\sqrt{5}+\sqrt{2})$

$= \sqrt{2}\times\sqrt{5}+\sqrt{2}\times\sqrt{2}+\sqrt{3}\times\sqrt{5}+\sqrt{3}\times\sqrt{2}$

$= \sqrt{10}+\sqrt{4}+\sqrt{15}+\sqrt{6}$

$= \sqrt{10}+2+\sqrt{15}+\sqrt{6}$ Ans.

iv) $(3-2\sqrt{3})(3+2\sqrt{3})$

Given $(3-2\sqrt{3})(3+2\sqrt{3})$

Use formula $(a-b)(a+b)=a^2-b^2$

$(3-2\sqrt{3})(3+2\sqrt{3}) = (3)^2-(2\sqrt{3})^2$

$= 9-2^2\times(\sqrt{3})^2$

$= 9-4\times3$

$= 9-12 = -3$ Ans.

Q4: Rationalize the denominator and simplify:

i) $\frac{1}{\sqrt{7}}$ ii) $\frac{3}{\sqrt{45}}$

iii) $\frac{1}{\sqrt{2}-1}$ iv) $\frac{5}{2+\sqrt{5}}$

v) $\frac{1}{\sqrt{5}-2}+\frac{1}{\sqrt{5}+2}$

Solution:

i) $\frac{1}{\sqrt{7}}$

Given $\frac{1}{\sqrt{7}}$

Divide and multiply by $\sqrt{7}$

$= \frac{1}{\sqrt{7}}\times\frac{\sqrt{7}}{\sqrt{7}} = \frac{\sqrt{7}}{\sqrt{7\times7}} = \frac{\sqrt{7}}{7}$ Ans.

ii) $\frac{3}{\sqrt{45}}$

Given $\frac{3}{\sqrt{45}}$

$$\frac{3}{\sqrt{45}}=\frac{3}{\sqrt{9\times5}}$$

$$=\frac{\cancel{3}}{\cancel{3}\sqrt{5}}=\frac{1}{\sqrt{5}}$$

$$=\frac{1}{\sqrt{5}}\times\frac{\sqrt{5}}{\sqrt{5}}$$

Divide and multiply by $\sqrt{5}$

$$=\frac{\sqrt{5}}{\sqrt{5}\times5}=\frac{\sqrt{5}}{5} \quad \text{Ans.}$$

iii) $\frac{1}{\sqrt{2}-1}$

Given $\sqrt{2}-1$

$$=\frac{1}{\sqrt{2}-1}\times\frac{\sqrt{2}+1}{\sqrt{2}+1}$$

(Divide and multiply by $\sqrt{2}+1$)

$$=\frac{\sqrt{2}+1}{(\sqrt{2}-1)(\sqrt{2}+1)}$$

$$=\frac{\sqrt{2}+1}{(\sqrt{2})^2-(1)^2}$$

$$\because (a-b)(a+b)=a^2-b^2$$

$$=\frac{\sqrt{2}+1}{2-1}=\frac{\sqrt{2}+1}{1}$$

$$=\sqrt{2}+1 \quad \text{Ans.}$$

iv) $\frac{5}{2+\sqrt{5}}$

Given $\frac{5}{2+\sqrt{5}}$

(Divide and multiply by $2-\sqrt{5}$)

$$=\frac{5}{2+\sqrt{5}}\times\frac{2-\sqrt{5}}{2-\sqrt{5}}$$

$$=\frac{5(2-\sqrt{5})}{(2+\sqrt{5})(2-\sqrt{5})}$$

$$=\frac{5(2-\sqrt{5})}{(2)^2-(\sqrt{5})^2} \quad \because (a-b)(a+b)=a^2-b^2$$

$$=\frac{5(2-\sqrt{5})}{4-5}$$

$$=\frac{5(2-\sqrt{5})}{-1}$$

$$=-5(2-\sqrt{5})$$

$$=-10+5\sqrt{5}$$

$$=5\sqrt{5}-10 \quad \text{Ans.}$$

v) $\frac{1}{\sqrt{5}-2}+\frac{1}{\sqrt{5}+2}$

Given $\frac{1}{\sqrt{5}-2}+\frac{1}{\sqrt{5}+2}$

By L.C.M

$$=\frac{\sqrt{5}+\cancel{2}+\sqrt{5}-\cancel{2}}{(\sqrt{5}-2)(\sqrt{5}+2)}$$

$$=\frac{2\sqrt{5}}{(\sqrt{5}-2)(\sqrt{5}+2)}$$

$$=\frac{2\sqrt{5}}{(\sqrt{5})^2-(2)^2}$$

$$=\frac{2\sqrt{5}}{5-4}=\frac{2\sqrt{5}}{1}$$

$$=2\sqrt{5} \quad \text{Ans.}$$

Q5: If $x=\sqrt{5}+2$, find the values of $x+\frac{1}{x}$ and $x^2+\frac{1}{x^2}$.

Solution:

Given $x=\sqrt{5}+2$

$x=\sqrt{5}+2$ then $\frac{1}{x}=\frac{1}{\sqrt{5}+2}$

(Divide and multiply by $\sqrt{5}-2$)

$$\frac{1}{x}=\frac{1}{\sqrt{5}+2}\times\frac{\sqrt{5}-2}{\sqrt{5}-2}$$

$$=\frac{\sqrt{5}-2}{(\sqrt{5}+2)(\sqrt{5}-2)}$$

$$=\frac{\sqrt{5}-2}{(\sqrt{5})^2-(2)^2}$$

$$=\frac{\sqrt{5}-2}{5-4}$$

$\frac{1}{x}=\frac{\sqrt{5}-2}{1}=\sqrt{5}-2$

Now $x+\frac{1}{x}=\sqrt{5}+\cancel{2}+\sqrt{5}-\cancel{2}$

$\therefore \quad =2\sqrt{5}$

$\boxed{x+\frac{1}{x}=2\sqrt{5}}$ Ans.

Square on both sides

$\left(x+\frac{1}{x}\right)^2=\left(2\sqrt{5}\right)^2$

$\Rightarrow x^2+\frac{1}{x^2}+2=4\times 5$

$\Rightarrow x^2+\frac{1}{x^2}+2=20$

$\Rightarrow x^2+\frac{1}{x^2}=20-2$

$\Rightarrow \boxed{x^2+\frac{1}{x^2}=18}$ Ans.

Q6: If $x=\sqrt{2}+\sqrt{3}$, find the values of $x-\frac{1}{x}$ and $x^2+\frac{1}{x^2}$.

Solution:

Given $x=\sqrt{2}+\sqrt{3}$

$x=\sqrt{2}+\sqrt{3}$ then $\frac{1}{x}=\frac{1}{\sqrt{2}+\sqrt{3}}$

(Divide and multiply by $\sqrt{2}-\sqrt{3}$)

$\frac{1}{x}=\frac{1}{\sqrt{2}+\sqrt{3}}\times\frac{\sqrt{2}-\sqrt{3}}{\sqrt{2}-\sqrt{3}}$

$=\frac{\sqrt{2}-\sqrt{3}}{(\sqrt{2}+\sqrt{3})(\sqrt{2}-\sqrt{3})}$

$=\frac{\sqrt{2}-\sqrt{3}}{(\sqrt{2})^2-(\sqrt{3})^2}$

$\frac{1}{x}=\frac{\sqrt{2}-\sqrt{3}}{2-3}=\frac{\sqrt{2}-\sqrt{3}}{-1}$

$=-1(\sqrt{2}-\sqrt{3})=-\sqrt{2}+\sqrt{3}$

Now $x-\frac{1}{x}=\left(\sqrt{2}+\sqrt{3}\right)-\left(-\sqrt{2}+\sqrt{3}\right)$

$x-\frac{1}{x}=\sqrt{2}+\cancel{\sqrt{3}}+\sqrt{2}-\cancel{\sqrt{3}}=2\sqrt{2}$

$\therefore \quad \boxed{x-\frac{1}{x}=2\sqrt{2}}$ Ans.

Squaring both sides

$\left(x-\frac{1}{x}\right)^2=\left(2\sqrt{2}\right)^2$

$x^2+\frac{1}{x^2}-2=4(2)$

$x^2+\frac{1}{x^2}-2=8$

$x^2+\frac{1}{x^2}=8+2=10$

$\Rightarrow \boxed{x^2+\frac{1}{x^2}=10}$ Ans.

Q7: If $x=5-2\sqrt{6}$, find the values of $x+\frac{1}{x}$ and $x^2+\frac{1}{x^2}$.

Solution:

Given $x=5-2\sqrt{6}$

(Divide and multiply by $5+2\sqrt{6}$)

$x=5-2\sqrt{6}$ then $\frac{1}{x}=\frac{1}{5-2\sqrt{6}}$

$\frac{1}{x}=\frac{1}{5-2\sqrt{6}}\times\frac{5+2\sqrt{6}}{5+2\sqrt{6}}$

$=\frac{5+2\sqrt{6}}{(5-2\sqrt{6})(5+2\sqrt{6})}$

$=\frac{5+2\sqrt{6}}{(5)^2-(2\sqrt{6})^2}$

$\frac{1}{x}=\frac{5+2\sqrt{6}}{25-24}=\frac{5+2\sqrt{6}}{1}=5+2\sqrt{6}$

$x+\frac{1}{x}=5-\cancel{2\sqrt{6}}+5+\cancel{2\sqrt{6}}=10$

$\therefore \quad \boxed{x+\frac{1}{x}=10}$ Ans.

Squaring both sides

$$\left(x+\frac{1}{x}\right)^2=(10)^2$$

$$\Rightarrow x^2+\frac{1}{x^2}+2=100$$

$$\Rightarrow x^2+\frac{1}{x^2}=100-2$$

$$\Rightarrow \boxed{x^2+\frac{1}{x^2}=98}\text{ Ans.}$$

Q8: If $x=\frac{1}{\sqrt{2}-1}$, find the values of $x-\frac{1}{x}$ and $x^2+\frac{1}{x^2}$.

Solution:

Given $x=\frac{1}{\sqrt{2}-1}$

$x=\frac{1}{\sqrt{2}-1}$ then $\frac{1}{x}=\sqrt{2}-1$

(Divide and multiply by $\sqrt{2}+1$)

$$x=\frac{1}{\sqrt{2}-1}\times\frac{\sqrt{2}+1}{\sqrt{2}+1}$$

$$=\frac{\sqrt{2}+1}{(\sqrt{2}-1)(\sqrt{2}+1)}$$

$$=\frac{\sqrt{2}+1}{(\sqrt{2})^2-(1)^2}$$

$$=\frac{\sqrt{2}+1}{2-1}$$

$$x=\frac{\sqrt{2}+1}{1}=\sqrt{2}+1$$

$$x=\sqrt{2}+1$$

Now $x-\frac{1}{x}=(\sqrt{2}+1)-(\sqrt{2}-1)$

$$=\cancel{\sqrt{2}}+1-\cancel{\sqrt{2}}+1$$

$$\therefore \boxed{x-\frac{1}{x}=2}\text{ Ans.}$$

Square on both sides

$$\left(x-\frac{1}{x}\right)^2=(2)^2$$

$$\Rightarrow x^2+\frac{1}{x^2}-2=4$$

$$\Rightarrow x^2+\frac{1}{x^2}=4+2$$

$$\Rightarrow \boxed{x^2+\frac{1}{x^2}=6}\text{ Ans.}$$

Q9: If $x=\sqrt{10}+3$, find the values of $x-\frac{1}{x}$ and $x^2+\frac{1}{x}$.

Solution:

Given $x=\sqrt{10}+3$

$x=\sqrt{10}+3$ then $\frac{1}{x}=\frac{1}{\sqrt{10}+3}$

Divide and multiply by $\sqrt{10}-3$)

$$\frac{1}{x}=\frac{1}{\sqrt{10}+3}\times\frac{\sqrt{10}-3}{\sqrt{10}-3}$$

$$=\frac{\sqrt{10}-3}{(\sqrt{10})^2-(3)^2}$$

$$=\frac{\sqrt{10}-3}{10-9}$$

$$\frac{1}{x}=\frac{\sqrt{10}-3}{1}=\sqrt{10}-3$$

Now $x-\frac{1}{x}=(\sqrt{10}+3)-(\sqrt{10}-3)$

$$=\cancel{\sqrt{10}}+3-\cancel{\sqrt{10}}+3$$

$$\therefore \boxed{x-\frac{1}{x}=6}\text{ Ans.}$$

Squaring both sides, we get

$$\left(x-\frac{1}{x}\right)^2=(6)^2$$

$$\Rightarrow x^2+\frac{1}{x^2}-2=36$$

$$\Rightarrow x^2+\frac{1}{x^2}=36+2=38$$

$\boxed{x^2+\frac{1}{x^2}=38}$ Ans.

Q10: If $x=2-\sqrt{3}$, find the value of $x^4+\frac{1}{x^4}$.

Solution:

Given $x=2-\sqrt{3}$

$x=2-\sqrt{3}$ then $\frac{1}{x}=\frac{1}{2-\sqrt{3}}$

Divide and multiply by $2+\sqrt{3}$)

$$\frac{1}{x}=\frac{1}{2-\sqrt{3}}\times\frac{2+\sqrt{3}}{2+\sqrt{3}}$$

$$=\frac{2+\sqrt{3}}{(2-\sqrt{3})(2+\sqrt{3})}$$

$$=\frac{2+\sqrt{3}}{(2)^2-(\sqrt{3})^2}$$

$$=\frac{2+\sqrt{3}}{1}=2+\sqrt{3}$$

$$\frac{1}{x}=\frac{2+\sqrt{3}}{4-3}$$

Now $x+\frac{1}{x}=2-\cancel{\sqrt{3}}+2+\cancel{\sqrt{3}}=4$

$\therefore\quad x+\frac{1}{x}=4$

Squaring both sides

$$\left(x+\frac{1}{x}\right)^2=(4)^2$$

$$\Rightarrow x^2+\frac{1}{x^2}+2=16$$

$$\Rightarrow x^2+\frac{1}{x^2}=16-2=14$$

$\therefore\quad x^2+\frac{1}{x^2}=14$

Square again both sides

$$\left(x^2+\frac{1}{x^2}\right)^2=(14)^2$$

$$x^4+\frac{1}{x^4}+2=196$$

$$x^4+\frac{1}{x^4}=196-2$$

$\Rightarrow x^4+\frac{1}{x^4}=194$ Ans.

Review Exercise 4

Q1: Select the correct answer.

i) $mr^2+3mr^2-5mr^2=$

✓ **(a) $-mr^2$** (b) $-mr$

(c) mr (d) mr^2

ii) $(x^3y^2)(x^2y^3)=$

✓ **(a) x^5y^5**

(b) x^5y^4

(c) x^4y^5

(d) x^4y^4

iii) $(4xy^4)^3=$

(a) $64x^3y^8$

(b) $64x^3y^{10}$

✓ **(c) $64x^3y^{12}$**

(d) $64x^3y^7$

iv) $(7x+4y)-(3x-6y)=$

(a) $3x$

(b) $2x+10y$

(c) $4x+y$

✓ **(d) $4x+10y$**

v) $(a+b)^2-(a-b)^2=$

✓ **(a) $4ab$**

(b) $2(a^2+b^2)$

(c) $a^2-4ab+2b^2$

(d) a^4-b^4

vi) $(a+b+c)^2=$

(a) $a^2+b^2+c^2$

(b) $a^2+b^2+c^2+2(a+b+c)$

✓ **(c) $a^2+b^2+c^2+2(ab+bc+ca)$**

(d) $a+b+c+2(ab+bc+ca)$

vii) $a^3+b^3=$

(a) $(a+b)^3-2ab(a+b)$

(b) $(a+b)(a^2+ab+b^2)$

(c) $(a-b)(a^2-ab+b^2)$

✓(d) $(a+b)(a^2-ab+b^2)$

viii) Conjugate of $3-\sqrt{5}$ is:

(a) $-3-\sqrt{5}$

(b) $-3+\sqrt{5}$

✓(c) $3+\sqrt{5}$

(d) $\sqrt{3}+\sqrt{5}$

ix) Which of the following expressions is equivalent to the expression $(m^2+4)^{-\frac{1}{2}}$?

(a) $\frac{(m^2+4)}{2}$

(b) $-\sqrt{m^2+4}$

✓(c) $\frac{1}{\sqrt{m^2+4}}$

(d) $\frac{1}{m+2}$

x) For which of the following expressions $a+b$ is not a factor?

(a) a^2-b^2

✓(b) a^2+b^2

(c) a^3+b^3

(d) a^4-b^4

Q2: Simplify: $\frac{12x^4y^5}{25a^3b^4}\cdot\frac{15a^5b^4}{16x^7y^2}$

Solution:

Given $\frac{12x^4y^5}{25a^3b^4}\cdot\frac{15a^5b^4}{16x^7y^2}$

$$\Rightarrow \frac{{}^3\cancel{12}x^4y^5}{\cancel{25}_5a^3\cancel{b^4}}\times\frac{\cancel{15}^3a^5\cancel{b^4}}{\cancel{16}_4x^7y^2}$$

$$=\frac{3x^4y^5}{5a^3}\times\frac{3a^5}{4x^7y^2}$$

$$=\frac{9}{20}a^{5-3}\frac{y^{5-2}}{x^{7-4}}$$

$$=\frac{9}{20}a^2\frac{y^3}{x^3} \quad \text{Ans.}$$

Q3: Evaluate $\frac{2x-3}{x^2-x+1}$ for $x=2$.

Solution:

Given $\frac{2x-3}{x^2-x+1}$

Putting the value $x=2$

$$=\frac{2(2)-3}{(2)^2-2+1}$$

$$=\frac{4-3}{4-2+1}=\frac{1}{3} \quad \text{Ans.}$$

Q4: Find the values of x^2+y^2 and xy, when $x+y=7$, $x-y=3$.

Solution:

We know from formula

$$2(x^2+y^2)=(x+y)^2+(x-y)^2$$

Putting the values in R.H.S

$$=(7)^2+(3)^2$$

$$=49+9=58$$

$$2(x^2+y^2)=58$$

Divide by 2 both sides

$$\frac{\cancel{2}(x^2+y^2)}{\cancel{2}}=\frac{\cancel{58}^{29}}{\cancel{2}}$$

$$\Rightarrow \boxed{x^2+y^2=29} \quad \text{Ans.}$$

We know from formula

$$4xy=(x+y)^2-(x-y)^2$$

Putting the values in R.H.S

$$=(7)^2-(3)^2$$

$$=49-9=40$$

$$4xy=40$$

(Divide by 4 both sides)

$$\frac{\cancel{4}xy}{\cancel{4}}=\frac{\cancel{40}^{10}}{\cancel{4}}$$

$$\Rightarrow \boxed{xy=10} \quad \text{Ans.}$$

Q5: Find the values of $a+b+c$ when $a^2+b^2+c^2=43$ & $ab+bc+ca=3$.

Solution:

We know from formula

$(a+b+c)^2=a^2+b^2+c^2+2ab+2bc+2ca$

$(a+b+c)^2=a^2+b^2+c^2+2(ab+bc+ca)$

Putting the values in R.H.S

$=43+2(3)$

$=43+6=49$

$(a+b+c)^2=49$

Taking square root of both sides

$\Rightarrow \sqrt{(a+b+c)^2}=\sqrt{49}$

$\Rightarrow a+b+c=\pm 7$ Ans.

Q6: If $a+b+c=6$ & $a^2+b^2+c^2=24$, then find the value of $ab+bc+ca$.

Solution:

Given $a+b+c=6$

Squaring both sides, we get

$(a+b+c)^2=(6)^2$

$a^2+b^2+c^2+2ab+2bc+2ca=36$

$a^2+b^2+c^2+2(ab+bc+ca)=36$

Putting the values

$24+2(ab+bc+ca)=36$

$2(ab+bc+ca)=36-24=12$

$2(ab+bc+ca)=12$ (Divide by 2)

$\frac{\cancel{2}(ab+bc+ca)}{\cancel{2}}=\frac{\cancel{12}^{6}}{\cancel{2}}$

$\Rightarrow \boxed{ab+bc+ca=6}$ Ans.

Q7: If $2x-3y=8$ and $xy=2$, then find the value of $8x^3-27y^3$.

Solution:

Given $2x-3y=8$ and $xy=2$

Taking cube on both sides

$(2x-3y)^3=(8)^3$

$\Rightarrow (2x)^3-(3y)^3-3(2x)(3y)(2x-3y)=512$

$\Rightarrow 8x^3-27y^3-18xy(2x-3y)=512$

Put values of $2x-3y=8$ and $xy=2$

$\Rightarrow 8x^3-27y^3-18(2)(8)=512$

$\Rightarrow 8x^3-27y^3-288=512$

$\Rightarrow 8x^3-27y^3=512+288=800$

$\Rightarrow \boxed{8x^3-27y^3=800}$ Ans.

Note: Book question # 7 is wrong

Q8: If $x+\frac{1}{x}=8$, then find the value of $x^3+\frac{1}{x^3}$.

Solution:

Given $x+\frac{1}{x}=8$

$x+\frac{1}{x}=8$

Taking cube on both sides, we get

$\Rightarrow \left(x+\frac{1}{x}\right)^3=(8)^3$

$x^3+\frac{1}{x^3}+3\left(x+\frac{1}{x}\right)=512$

Put values of $x+\frac{1}{x}=8$

$x^3+\frac{1}{x^3}+3(8)=512$

$\Rightarrow x^3+\frac{1}{x^3}+24=512$

$x^3+\frac{1}{x^3}=512-24$

$$\Rightarrow x^3+\frac{1}{x^3}=488$$

$$\therefore \boxed{x^3+\frac{1}{x^3}=488} \quad \text{Ans.}$$

Q9: Find the product:

$$\left(\frac{4}{5}x-\frac{5}{4x}\right)\left(\frac{16}{25}x^2+\frac{25}{16x^2}+1\right)$$

Solution:

Given $\left(\frac{4}{5}x-\frac{5}{4x}\right)\left(\frac{16}{25}x^2+\frac{25}{16x^2}+1\right)$

$$\left(\frac{4}{5}x-\frac{5}{4x}\right)\left[\left(\frac{4}{5}x\right)^2+\left(\frac{4}{5}x\right)\left(\frac{5}{4x}\right)+\left(\frac{5}{4x}\right)^2\right]$$

Formula; $(a-b)(a^2+ab+b^2)=a^3-b^3$

$$=\left(\frac{4}{5}x\right)^3-\left(\frac{5}{4x}\right)^3$$

$$=\frac{64}{125}x^3-\frac{125}{64x^3} \quad \text{Ans.}$$

Think:

Q10: Simplify $\frac{2x^2}{x^4-16}-\frac{x}{x^2-4}+\frac{1}{x+2}$

Solution:

Given $\frac{2x^2}{x^4-16}-\frac{x}{x^2-4}+\frac{1}{x+2}$

$$=\frac{2x^2}{(x^2+4)(x^2-4)}-\frac{x}{x^2-4}+\frac{1}{x+2}$$

Taking L.C.M of first two terms

$$=\frac{2x^2}{(x^2+4)(x^2-4)}-\frac{x}{x^2-4}+\frac{1}{x+2}$$

$$=\frac{2x^2-x(x^2+4)}{(x^2+4)(x^2-4)}+\frac{1}{x+2}$$

$$=\frac{2x^2-x^3-4x}{(x^2+4)(x+2)(x-2)}+\frac{1}{x+2}$$

$$=\frac{2x^2-x^3-4x+(x^2+4)(x-2)}{(x^2+4)(x+2)(x-2)}$$

$$=\frac{\cancel{2x^2}-\cancel{x^3}-\cancel{4x}+\cancel{x^3}-\cancel{2x^2}+\cancel{4x}-8}{(x^2+4)(x^2-4)}$$

$$=\frac{-8}{(x^2+4)(x^2-4)}$$

$$=\frac{-8}{(x^2)^2-(4)^2} \quad \because (a+b)(a-b)=a^2-b^2$$

$$=\frac{-8}{x^4-16}$$

$$=-\frac{-8}{x^4-16} \quad \text{Ans.}$$

***

Additional MCQs of Unit 4:
Algebraic Expressions & Algebraic Formulas

1. In polynomials $P(x)$ the power is..........
   (a) Real number (b) Natural number (c) Non-negative number (d) none
   ✓ **Ans. (c) Non-negative number**
2. $(a+b)^2+(a-b)^2=$ ....
   (a) a^2+b^2 (b) $4ab$ (c) $2(a^2+b^2)$ (d) none
   ✓ **Ans. (c) $2(a^2+b^2)$**
3. $(a+b)^2-(a-b)^2=$ ....
   (a) $2(a^2+b^2)$ (b) a^2+b^2 (c) $4ab$ (d) $-4ab$
   ✓ **Ans. (c) $4ab$**
4. $\dfrac{x^2+3}{x}$ if $x\neq 0$ is..........expression.
   (a) Rational (b) Irrational (c) Polynomial (d) none
   ✓ **Ans. (a) Rational**
5. $\dfrac{x+2}{x^2-4}$ its lowest term is..........
   (a) $\dfrac{1}{x-2}$ (b) $\dfrac{1}{x+2}$ (c) $\dfrac{x+2}{x-2}$ (d) none
   ✓ **Ans. (b) $\dfrac{1}{x+2}$**
6. When $x=2$, $y=-3$ then $x^2-y^2=$ ....
   (a) 6 (b) 5 (c) 9 (d) -5
   ✓ **Ans. (d) -5**
7. $a^3+3ab(a+b)+b^3=$ ....
   (a) $(a+b)^3$ (b) $(a-b)^3$ (c) a^3+b^3 (d) none
   ✓ **Ans. (a) $(a+b)^3$**
8. $(a-b)(a^2+ab+b^2)=$ ....
   (a) a^3+b^3 (b) a^3-b^3 (c) $(a+b)^3$ (d) $(a-b)^3$
   ✓ **Ans. (b) a^3-b^3**
9. A number of the form $\sqrt[n]{a}$ where $a>0$ is called..........
   (a) Radical (b) Radicand (c) Surd (d) none
   Ans. (c) Surd
10. If $x=2+\sqrt{3}$ then $\dfrac{1}{x}=$ ....
    (a) $\dfrac{1}{2+\sqrt{3}}$ (b) $2-\sqrt{3}$ (c) $\dfrac{1}{2-\sqrt{3}}$ (d) none
    ✓ **Ans. (b) $2-\sqrt{3}$**

UNIT 6:
ALGEBRAIC MANIPULATION

Highest Common Factor (H.C.F):

Definition: The highest number of factors common to all given expressions or polynomials is known as *highest common factor (H.C.F)*.

EXAMPLE 1

Find H.C.F of $x^2 - y^2$, $x^2 - xy$

Solution:

Given that

$x^2 - y^2 = (x+y)(x-y)$

$x^2 - xy = x(x-y)$

We see that $x - y$ is a common factor.

$\therefore$ H.C.F $= x - y$

There are two methods to solve or find H.C.F:

1. H.C.F by factorization
2. H.C.F by division

Steps to find H.C.F by division method:

i) Write the given expression in descending order with respect to variable.
ii) Divide higher degree polynomial by lower degree polynomial.
ii) Repeat the process till the remainder is zero.
v) Last divisor is the required divisor H.C.F of the given polynomials.

Vote: When we want to find H.C.F of three polynomials, first find H.C.F of any two of hem, then find H.C.F of this H.C.F and he third polynomial.

EXAMPLE 2

ind H.C.F of $ax^2 + 5ax + 6a$, $ax^3 +$ $ax^2 + 14ax$ and $15a(x^2 - 4)$.

lution:

$+ 5ax + 6a = a(x^2 + 5x + 6)$

$= a(x^2 + 3x + 2x + 6)$

$= a[x(x+3) + 2(x+3)]$

$= a(x+3)(x+2)$

$ax^3 + 9ax^2 + 14ax = ax(x^2 + 9x + 14)$

$= ax(x^2 + 7x + 2x + 14)$

$= ax[x(x+7) + 2(x+7)]$

$= ax(x+7)(x+2)$

$15a(x^2 - 4) = 15a(x-2)(x+2)$

In the above factorization we see that $a(x+2)$ is common to given three expressions, it also divides exactly the given expressions.

$\therefore$ H.C.F $= a(x+2)$

H.C.F by Division:

EXAMPLE 3

Find H.C.F of $2x^3 + 7x^2 + 4x - 4$ and $2x^3 + 9x^2 + 11x + 2$.

Solution:

Let $P(x) = 2x^3 + 7x^2 + 4x - 4$ and $Q(x) =$ $2x^3 .9x^2 + 11x + 2$

$$2x^3+7x^2+4x-4 \,\big)\, \cancel{2x^3} + 9x^2 + 11x + 2 \,\big(\, 1$$
$$\pm\cancel{2x^3} \pm 7x^2 \pm 4x \mp 4$$
$$2x^2+7x+6 \,\big)\, \cancel{2x^3} + \cancel{7x^2} + 4x - 4 \,\big(\, x$$
$$\pm\cancel{2x^3} \pm \cancel{7x^2} \pm 6x$$
$$-2 \,\big|\, -2x - 4 \quad \text{Dividing by -2}$$
$$x+2 \,\big)\, \cancel{2x^2} + 7x + 6$$
$$\pm\cancel{2x^2} \pm 4x$$
$$\cancel{3x} + \cancel{6}$$
$$\pm\cancel{3x} \pm \cancel{6}$$
$$\times$$

Least Common Multiple (L.C.M):

The L.C.M of two or more polynomials is the polynomial of least degree which is divisible by the given polynomials. We find L.C.M by two methods.

1. L.C.M by factorization
2. L.C.M by division

EXAMPLE 5

Find L.C.M of x^2+4x+4 and x^2+5x+6.

Solution:

$x^2+4x+4=(x+2)^2$
$=(x+2)(x+2)$

$x^2+5x+6=x^2+3x+2x+6$
$=x(x+3)+2(x+3)$
$=(x+3)(x+2)$

L.C.M = common factor × non-common factor

L.C.M $=(x+2)\times(x+2)(x+3)$

L.C.M $=(x+2)^2(x+3)$

EXAMPLE 6

Find L.C.M of x^2-4x+3, x^2-3x+2 and x^2-5x+6.

Solution:

$x^2-4x=3=x^2-3x-x+3$
$=x(x-3)-1(x-3)$
$=(x-3)(x-1)\longrightarrow(1)$

$x^2-3x+2=x^2-2x-x+2$
$=x(x-2)-1(x-2)$
$=(x-2)(x-1)\longrightarrow(2)$

$x^2-5x+6=x^2-3x-2x+6$
$=x(x-3)-2(x-3)$
$=(x-3)(x-2)\longrightarrow(3)$

In expression (1) and (2) $x-1$ is a common factor.

In expression (2) and (3) $x-2$ is a common factor.

In expression (1) and (3) $x-3$ is a common factor.

Therefore

L.C.M = Common factor × Non-common factor
$=(x-1)(x-2)(x-3)\times1$
$=(x-1)(x-2)(x-3)$

Relationship between H.C.F and L.C.M:

Product of two polynomials
= H.C.F × L.C.M

Theorem

If A and B are given polynomials and their H.C.F and L.C.M are represented by H and L respectively then;

$A\times B=H\times L$

Proof:

Since H (H.C.F) is common factor of polynomials A and B, then it divides exactly A and B.

Let $\dfrac{A}{H}=a\longrightarrow(1)$

And $\dfrac{B}{H}=b\longrightarrow(2)$

Clearly a and b have no common factors.

Equation (1) and (2) can be written as

$A=Ha\longrightarrow(3)$

$B=Hb\longrightarrow(4)$

The L.C.M = common factor × non-common factor

Or $L=H\times a\times b$

Multiply both sides by H

$H\times L=H(H\times a\times b)$

Or $H\times L=H\times a\times H\times b$

Or $H\times L=(Ha)\times(Hb)$

Or $H\times L=A\times B$

Using equation (3) and (4)

Qr $\boxed{A\times B=H\times L}$

EXAMPLE 7

Find L.C.M of $x^3-6x^2+11x-6$ and x^3-4x+3.

Solution:

First we shall find H.C.F of given polynomials:

$x^3-4x+3\overline{)\cancel{x^3}-6x^2+11x-6}\,(1$

$\pm\cancel{x^3}\quad \pm 4x\pm 3$

$-3\,|\,-6x^2+15x-9$ Divide by -3

$2x^2-5x+3\overline{)x^3-4x+3}\,(x+5$

$\times 2$ Multiply by 2

$\cancel{2x^3}-8x+6$

$\pm\cancel{2x^3}\pm 3x \quad \pm 5x^2$

$5x^2-11x+6$

$\times 2$ Multiply by 2

$\cancel{10x^2}-22x+12$

$\pm\cancel{10x^2}\mp 25x\pm 15$

$3\,|\,3x-3$ Divide by 3

$x-1\overline{)\cancel{2x^2}-5x+3}\,(2x-3$

$\pm\cancel{2x^2}\mp 2x$

$-\cancel{3x}+\cancel{3}$

$\mp\cancel{3x}\pm\cancel{3}$

$\times$

Hence H.C.F of $x^3-6x^2+11x-6$ and x^3-4x+3 is $x-1$.

$\therefore \text{L.C.M} = \dfrac{\textit{product of two polynomials}}{H.C.F}$

$= \dfrac{(x^3-4x+3)(x^3-6x^2+11x-6)}{x-1}$

Divide $x^3-6x^2+11x-6$ by $x-1$

$x-1\overline{)\cancel{x^3}-6x^2+11x-6}\,(x^2-5x+6$

$\pm\cancel{x^3}\mp x^2$

$-\cancel{5x^2}+11x$

$\mp\cancel{5x^2}\pm 5x$

$\cancel{6x}-\cancel{6}$

$\pm\cancel{6x}\mp\cancel{6}$

0

Therefore L.C.M $=(x^3-4x+3)(x^2-5x+6)$

EXERCISE 6.1

Q1: Find H.C.F of the following expressions by factorization method.

i) $(x+6)^2$ and x^2-36

ii) x^4-y^4 and $x^4+2x^2y^2+y^4$

iii) $x-3,\ x^2-9,(x-3)^2$

iv) $2^3 3^2(x-y)^3(x+2y)^2,\ 2^3 3^2(x-y)^2(x+2y)^3, 3^2(x-y)^2(x+2y)$

v) $2x^4-2y^4, 6x^2+12xy+12xy+6y^2, 9x^3+9y^3$

Solution:

i) $(x+6)^2$ and x^2-36

Given $(x+6)^2=(x+6)(x+6)\rightarrow(1)$

And $x^2-36=(x^2)-(6)^2$

$=(x+6)(x-6)\rightarrow(2)$

From (1) and (2) $(x+6)$ is common.

Hence H.C.F $=\boxed{x+6}$ Ans.

ii) x^4-y^4 and $x^4+2x^2y^2+y^4$

Given $x^4-y^4=(x^2)^2-(y^2)^2$

$=(x^2-y^2)(x^2+y^2)$

$=(x+y)(x-y)(x^2+y^2)\rightarrow(1)$

And $x^4+2x^2y^2+y^4=(x^2)^2+2(x^2)(y^2)+(y^2)^2$

$=(x^2+y^2)^2=(x^2+y^2)(x^2+y^2)\rightarrow(2)$

From (1) and (2) x^2+y^2 is common factor

Hence H.C.F $=\boxed{x^2+y^2}$ Ans.

iii) $x-3,\ x^2-9,(x-3)^2$

Given $x-3=x-3\rightarrow(1)$

And $x^2-9=(x)^2-(3)^2$

$=(x-3)(x+3)\rightarrow(2)$

And $(x-3)^2=(x-3)(x-3)\rightarrow(3)$

From (1), (2) & (3) $x-3$ is common factor

Hence H.C.F $=\boxed{x-3}$ Ans.

iv) $2^3 3^2(x-y)^3(x+2y)^2,\ 2^3 3^2(x-y)^2(x+2y)^3, 3^2(x-y)^2(x+2y)$

Given $2^3 3^2(x-y)^3(x+2y)^2$

$=2.2.2.3.3(x-y)(x-y)(x-y)(x+2y)$

$(x+2y)\rightarrow(1)$

Now $2^3 3^2(x-y)^2(x+2y)^3$

$=2.2.2.3.3(x-y)(x-y)(x+2y)$

$(x+2y)(x+2y)\rightarrow(2)$

And $3^2(x-y)^2(x+2y) = 3.3(x-y)(x-y)(x+2y) \to (3)$

From (1), (2) and (3), H.C.F

$= 3.3.(x-y)(x-y)(x+2y)$

$= 3^2(x-y)^2(x+2y)$ Ans.

v) $2x^4-2y^4, 6x^2+12xy+12xy+6y^2, 9x^3+9y^3$

Given $2x^4-2y^4 = 2\left((x^2)^2-(y^2)^2\right)$

$= 2\left(x^2-y^2\right)\left(x^2+y^2\right)$

$= 2\left(x-y\right)\left(x+y\right)\left(x^2+y^2\right) \to (1)$

And $6x^2+12xy+6y^2 = 6x^2+6xy+6xy+6y^2$

$= 6x\left(x+y\right)+6y\left(x+y\right)$

$= \left(x+y\right)\left(6x+6y\right)$

$= 6\left(x+y\right)\left(x+y\right) \to (2)$

And $9x^3+9y^3 = 9\left(x^3+y^3\right)$

$= 9\left(x+y\right)\left(x^2-xy+y^2\right) \to (3)$

From (1), (2) and (3), the common factor is $x+y$

H.C.F $= \boxed{x+y}$ Ans.

Q2: Find H.C.F by division method.

i) x^2-x-6 and x^2-2x-3

ii) y^3-3y+2 and y^3-5y^2+7y-3

iii) $2x^5-4x^4-6x$ & $x^5+x^4-3x^3-3x^2$

iv) $2x^3+10x^2+5x+25$ and x^3+5x^2-x-5

Solution:

i) x^2-x-6 **and** x^2-2x-3

$x^2-x-6\,\overline{)\,x^2-2x-3\,}\,(1$

$\pm x^2 \mp x \mp 6$

$-\,|\,-x+3$

$x-3\,\overline{)\,x^2-x-6\,}\,(x+2$

$\pm x^2 \mp 3x$

$2x-6$

$\pm 2x \mp 6$

0

Hence required H.C.F $= \boxed{x-3}$ Ans.

ii) y^3-3y+2 **and** y^3-5y^2+7y-3

$y^3-3y+2\,\overline{)\,\cancel{y^3}-5y^2+7y-3\,}\,(1$

$\pm \cancel{y^3} \quad \mp 3y \pm 2$

Divide by -5 $|\,-5y^2+10y-5$

$y^2-2y+1\,\overline{)\,\cancel{y^3}-3y+2\,}\,(y+2$

$\pm \cancel{y^3} \pm y \quad \mp 2y^2$

$\cancel{2y^2}-\cancel{4y}+\cancel{2}$

$\pm \cancel{2y^2} \mp \cancel{4y} \pm \cancel{2}$

0

Hence required H.C.F

$= \boxed{y^2-2y+1}$ Ans.

iii) $2x^5-4x^4-6x$ & $x^5+x^4-3x^3-3x^2$

$x^5+x^4-3x^3-3x^2\,\overline{)\,\cancel{2x^5}-4x^4-6x\,}\,(2$

$\pm \cancel{2x^5} \pm 2x^4 \quad \mp 6x^3 \mp 6x^2$

Divide by -6 $|\,-6x^4+6x^3+6x^2-6x$

$x^4-x^3-x^2+x$

$x^4-x^3-x^2+x\,\overline{)\,\cancel{x^5}+x^4-3x^3-3x^2\,}\,(x+1$

$\pm \cancel{x^5} \mp x^4 \mp x^3 \pm x^2$

Divide by 2 $|\,2x^4-2x^3-4x^2$

$\cancel{x^4}-\cancel{x^3}-2x^2$

$\pm \cancel{x^4} \mp \cancel{x^3} \mp x^2 \pm x$

$-x^2-x\,\overline{)\,\cancel{x^4}-x^3-x^2+x\,}\,(-x^2+2x$

$\mp \cancel{x^4} \mp x^3$

$-\cancel{2x^3}-x^2+x$

$\mp \cancel{2x^3} \mp 2x^2$

$x^2+x\,\overline{)\,-\cancel{x^2}-\cancel{x}\,}$

$\mp \cancel{x^2} \mp \cancel{x}$

0

Hence required H.C.F

$= x^2+x$ or $x(x+1)$ Ans.

iv) $2x^3+10x^2+5x+25$ **and** x^3+5x^2-x-5

$$x^3+5x^2-x-5\overline{\big)\ \cancel{2x^3}+\cancel{10x^2}+5x+25}\ 2$$
$$\pm\cancel{2x^3}\pm\cancel{10x^2}\mp 2x\mp 10$$

Divide by 7 $\quad 7x+35$

$$x+5\ \big)\ \cancel{x^3}+\cancel{5x^2}-x-5\ \big(\ x^2$$
$$\pm\cancel{x^3}\pm\cancel{5x^2}$$

Divide by -1 $\quad -x-5$

$$x+5\ \big)\ \cancel{x}+\cancel{5}\ \big(\ 1$$
$$\pm\cancel{x}\pm\cancel{5}$$
$$0$$

Hence required H.C.F = $\boxed{x+5}$ Ans.

Q3: Find L.C.M by factorization:

i) $x+y,\ x^2-y^2$

ii) $x^3-y^3,\ x-y$

iii) $x^4-x,\ x^5-x^2$ and x^5-x^3

iv) $2^3 3^2(x-y)^3(x+2y)^2, 2^3 3^2(x-y)^2(x+2y)^3$ and $x^3+5x^2-x-5.$

Solution:

i) $x+y,\ x^2-y^2$

Given $x+y$ and $x^2-y^2=(x+y)(x-y)$

L.C.M = common factor × uncommon factor

$\therefore$ L.C.M $=(x+y)(x-y)$ Ans.

ii) $x^3-y^3,\ x-y$

Given $x^3-y^3=(x-y)(x^2+xy+y^2)$

And $(x-y)$

$\therefore$ L.C.M = common factor × un-common factor

L.C.M $=(x-y)(x^2+xy+y^2)$

Or L.C.M $=x^3-y^3$ Ans.

iii) $x^5-x,\ x^5-x^2$ and x^5-x^3

Given x^5-x, x^5-x^2, x^5-x^3

$x^5-x=x(x^4-1)$

$=x\left((x^2)^2-(1)^2\right)$

$=x(x^2-1)(x^2+1)$

$=x(x-1)(x+1)(x^2+1)$

And $x^5-x^2=x^2(x^3-1)$

$=x^2\left((x)^3-(1)^3\right)=x^2(x-1)(x^2+x+1)$

And $x^5-x^3=x^3(x^2-1)$

$=x^3(x-1)(x+1)$

$\therefore$ L.C.M = common factor × un-common factor

L.C.M $=x^3(x+1)(x-1)$

$(x^2+1)(x^2+x+1)$ Ans.

iv) $2^3 3^2(x-y)^3(x+2y)^2, 2^3 3^2(x-y)^2(x+2y)^3$ **and** $x^3+5x^2-x-5.$

Given $2^3 3^2(x-y)^3(x+2y)^2$

$=2.2.2.3.3(x-y)(x-y)(x-y)(x+2y)$

$(x+2y)\rightarrow(1)$

And $2^3 3^2(x-y)^2(x+2y)^3$

$=2.2.2.3.3(x-y)(x-y)(x+2y)$

$(x+2y)(x+2y)\rightarrow(2)$

And given $3^2(x-y)^2(x+2y)$

$=3.3(x-y)(x-y)(x+2y)\rightarrow(3)$

From (1), (2) and (3), L.C.M

$=2.2.2.3.3(x-y)(x-y)(x-y)$

$(x+2y)(x+2y)(x+2y)$

$=2^3 3^2(x-y)^3(x+2y)^3$ Ans.

Q4: Find H.C.F and L.C.M of the following expressions:

i) $x^3-2x^2-13x-10$ & $x^3-x^2-10x-8$

ii) $2x^4-2x^3+x^2+3x-6$ and $4x^4-2x^3+3x-9$

iii) a^4-a^3-a+1 and a^4+a^2+1

iv) $1-x^2-x^4+x^5$ and $1+2x+x^2-x^4-x^5$

Solution:

i) $x^3-2x^2-13x-10$ & $x^3-x^2-10x-8$

Let $A=x^3-2x^2-13x-10,$

$B=x^3-x^2-10x-8$

First to find H.C.F

$$x^4-x^2-10x-8 \overline{) x^4-2x^2-13-10} \ (1$$
$$\pm x^4 \mp x^2 \mp 10x \mp 8$$
$$\boxed{-x^2-3x-2}$$
$$-1 \big| x^2+3x+2 \overline{) x^3-x^2-10x-8} \ (x$$
$$-4x^2-12x-8$$
Divide by -4 $\boxed{-4x^2-12x-8} \quad -1$
$$x^2+3x+2 \overline{) \cancel{x^2}+\cancel{3x}+\cancel{2}}$$
$$\pm\cancel{x^2}\pm\cancel{3x}\pm\cancel{2}$$
$$0$$

$\therefore$ H.C.F $= x^2+3x+2$

$$\text{L.C.M} = \frac{A \times B}{H.C.F} \rightarrow (1)$$

Put values

$$= \frac{(x^3-2x^2-13x-10)(x^3-x^2-10x-8)}{x^2+3x+2}$$

Divide B by H.C.F

$$\begin{array}{r} x-4 \\ x^2+3x+2 \overline{) \cancel{x^3}-x^2-10x-8} \\ \pm\cancel{x^3} \pm 3x^2 \pm 2x \\ \hline -\cancel{4x^2}-\cancel{12x}-\cancel{8} \\ \mp\cancel{4x^2}\mp\cancel{12x}\mp\cancel{8} \\ \hline 0 \end{array}$$

Hence required L.C.M

$$= (x-4)(x^3-2x^2-13x-10) \text{ Ans.}$$

ii) $2x^4-2x^3+x^2+3x-6$ **and** $4x^4-2x^3+3x-9$

Let $A = 2x^4-2x^3+x^2+3x-6$

And $B = 4x^4-2x^3+3x-9$

First find H.C F

$$2x^4-2x^3+x^2+3x-6 \overline{) \cancel{4x^4}-2x^3+3x-9} \ (2$$
$$\cancel{4x^4} \mp 4x^3 \pm 6x \mp 12 \pm 2x^2$$
$$2x^3-2x^2-3x+3 \overline{) \cancel{2x^4}-\cancel{2x^3}+x^2+\cancel{3x}-6} \ (x$$
$$-\cancel{2x^4} \mp \cancel{2x^3} \mp 3x^2 \pm \cancel{3x}$$
Divide by 2 $\big| 4x^2-6$
$$2x^2-3 \big| \cancel{2x^3}-2x^2-\cancel{3x}+3 \ (x-1$$
$$\pm\cancel{2x^3} \quad \mp\cancel{3x}$$
$$-\cancel{2x^2}+\cancel{3}$$
$$\mp\cancel{2x^2}\pm\cancel{3}$$
$$0$$

Hence required H.C.F

$$= H = \boxed{2x^2-3} \quad \text{Ans.}$$

$$\text{L.C.M} = \frac{A \times B}{H.C.F} \rightarrow (1)$$

Put values in (1)

$$= \frac{(2x^4-2x^3+x^2+3x+6)(4x^4-2x^3+3x-9)}{2x^2-3}$$

Divide A by H.C.F

$$\begin{array}{r} x^2-x+2 \\ 2x^2-3 \overline{) \cancel{2x^4}-2x^3+x^2+3x-6} \\ \pm\cancel{2x^4} \qquad \mp 3x^2 \\ \hline -\cancel{2x^3}+4x^2+3x-6 \\ \mp\cancel{2x^3} \qquad \pm 3x \\ \hline \cancel{4x^2}-\cancel{6} \\ \pm\cancel{4x^2}\mp\cancel{6} \\ \hline 0 \end{array}$$

Hence required L.C.M

$$= (x^2-x+2)(4x^4-2x^3+3x-9) \text{ Ans.}$$

iii) a^4-a^3-a+1 **and** a^4+a^2+1

Let $A = a^4-a^3-a+1$

And $B = a^4+a^2+1$

First find H.C.F

$$a^4+a^2+1 \overline{) \cancel{a^4}-a^3-a+\cancel{1}} \ (1$$
$$\cancel{a^4} \quad \pm\cancel{1} \quad \pm a^2$$
Divide by -1 $\boxed{-a^3-a^2-a}$
$$a^3+a^2+a \overline{) \cancel{a^4}+\cancel{a^2}+1} \ (a$$
$$\pm\cancel{a^4}\pm\cancel{a^3} \quad \pm a^3$$
$$-a^3+1 \overline{) \cancel{a^3}+a^2+a} \ (-1$$
$$\pm\cancel{a^3} \quad \mp 1$$
$$a^2+a+1 \overline{) -\cancel{a^3}+1} \ (-a$$
$$\mp\cancel{a^3}\mp a^2 \mp a$$
$$a^2+a+1$$

$$\begin{array}{r} 1 \\ a^2+a+1 \overline{) \cancel{a^2}+\cancel{a}+\cancel{1}} \\ \pm\cancel{a^2}\pm\cancel{a}\pm\cancel{1} \\ \hline 0 \end{array}$$

$\boxed{H.C.F = a^2 + a + 1}$ Ans.

$$L.C.M = \frac{A\times B}{H.C.F} \to (1)$$

Put values in (1)

$$= \frac{(a^4 - a^3 - a + 1)(a^4 + a^2 + 1)}{a^2 + a + 1}$$

Divide A by H.C.F

$$\begin{array}{r|l} & a^2 - 2a + 1 \\ a^2 + a + 1 & \cancel{a^4} - a^3 - a + 1 \\ & \pm a^4 \pm a^3 \quad \pm a^2 \\ \hline & -\cancel{2a^3} - a^2 - a + 1 \\ & \mp \cancel{2a^3} \mp 2a^2 \mp 2a \\ \hline & \cancel{a^2} + \cancel{a} + \cancel{1} \\ & \pm \cancel{a^2} \pm \cancel{a} \pm \cancel{1} \\ \hline & 0 \end{array}$$

$\therefore$ Required L.C.M

$= (a^2 - 2a + 1)(a^4 + a^2 + 1)$ Ans.

iv) $1 - x^2 - x^4 + x^5$ & $1 + 2x + x^2 - x^4 - x^5$

Let $A = 1 - x^2 - x^4 + x^5$

And $B = 1 + 2x + x^2 - x^4 - x^5$

We find H.C.F

$$\begin{array}{r|l} x^5 - x^4 - x^2 + 1 & -\cancel{x^5} - x^4 + \cancel{x^2} + 2x + 1 \quad (-1 \\ & \mp \cancel{x^5} \pm x^4 \pm \cancel{x^2} \\ \hline \text{Taking -2 common} & -2x^4 + 2x + 2 \\ \hline x^4 - x - 1 & x^5 - x^4 - x^2 - 1 \quad (x - 1 \\ & \pm x^5 \quad \mp x^2 \quad \mp x \\ \hline & -\cancel{x^4} + \cancel{x} + \cancel{1} \\ & \mp \cancel{x^4} \pm \cancel{x} \pm \cancel{1} \\ \hline & 0 \end{array}$$

$\therefore$ $\boxed{H.C.F = x^4 - x - 1}$ Ans.

$$L.C.M = \frac{A\times B}{H.C.F} \to (1)$$

$$L.C.M = \frac{(x^5 - x^4 - x^2 + 1)(-x^5 - x^4 + x^2 + 2x + 1)}{x^4 - x - 1}$$

$$\begin{array}{r|l} & x - 1 \\ x^4 - x - 1 & \cancel{x^5} - x^4 - \cancel{x^2} + 1 \\ & \pm \cancel{x^5} \quad \mp \cancel{x^2} \quad \mp x \\ \hline & -\cancel{x^4} + \cancel{x} + \cancel{1} \\ & \mp \cancel{x^4} \pm \cancel{x} \pm \cancel{1} \\ \hline & 0 \end{array}$$

$\therefore L.C.M = (x - 1)(-x^5 - x^4 + x^2 + 2x + 1)$

$\Rightarrow L.C.M = (x - 1)(1 + 2x + x^2 - x^4 - x^5)$

Q5: H.C.F and L.C.M of two polynomials are x-2 and $x^3 + 3x^2 - 6x - 8$ respectively. If one polynomial is $x^2 + 2x - 8$, find the second polynomial.

Solution:

Given $H = x - 2$

$L.C.M = x^3 + 3x^2 - 6x - 8$

And $A = x^2 + 2x - 8$ then we find B

$A \times B = H \times L$

$$\Rightarrow B = \frac{H \times L}{A} \longrightarrow (1)$$

$$= \frac{(x - 2)(x^3 + 3x^2 - 6x - 8)}{x^2 + 2x - 8}$$

$$\begin{array}{r|l} & x + 1 \\ x^2 + 2x - 8 & \cancel{x^3} + 3x^2 - 6x - 8 \\ & \pm \cancel{x^3} \pm 2x^2 \mp 8x \\ \hline & \cancel{x^2} + \cancel{2x} - \cancel{8} \\ & \pm \cancel{x^2} \pm \cancel{2x} \pm \cancel{8} \\ \hline & 0 \end{array}$$

Now $B = (x - 2)(x + 1)$

$B = x^2 + x - 2x - 2$

$\Rightarrow \boxed{B = x^2 - x - 2}$ Ans.

Q6: If product of two polynomials is $x^4 + 5x^3 - 6x^2 - 2x - 28$ and their H.C.F is x-2. Find their L.C.M.

Solution:

Given $A \times B = x^4 + 5x^3 - 6x^2 - 2x - 28$

$H = x - 2$ to find $L = ?$

$$\text{As } L = \frac{A \times B}{H} \longrightarrow (1)$$

$$=\frac{x^4+5x^3-6x^2-2x-28}{x-2}$$

$$
\begin{array}{r}
x^3+7x^2+8x+14 \\
x-2\,\big)\;\cancel{x^4}+5x^3-6x^2-2x-28 \\
\pm\cancel{x^4}\mp 2x^3 \\
\hline
\cancel{7x^3}-6x^2-2x-28 \\
\pm\cancel{7x^3}\mp 14x^2 \\
\hline
\cancel{8x^2}-2x-28 \\
\pm\cancel{8x^2}\mp 16x \\
\hline
\cancel{14x}-\cancel{28} \\
\pm\cancel{14x}\mp\cancel{28} \\
\hline
0
\end{array}
$$

$\therefore$ L.C.M $=\boxed{x^3+7x^2+8x+14}$ Ans.

Q7: H.C.F and L.C.M of two polynomials are $x+5$ and $2x^3+11x^2+2x-15$ respectively. Find polynomials of degree 2.

Solution:

Given $H=x+5$

$L=2x^3+11x^2+2x-15$

We find both polynomials A and B

As $A\times B=H\times L$

$=(x+5)(2x^3+11x^2+2x-15)$

$x=1,\ x=r=1$ Put in $P(x)$

$P(1)=2(1)^3+11(1)^2+2(1)-15$

$=2+11+2-15=0$

$\therefore x=1$ or $x-1$ is one factor. For 2nd factor divide L by $x-1$

$$
\begin{array}{r}
2x^2+13x+15 \\
x-1\,\big)\;2x^3+11x^2+2x-15 \\
2x^3\mp 2x^2 \\
\hline
\cancel{13x^2}+2x \\
\cancel{13x^2}\mp 13x \\
\hline
\cancel{15x-15} \\
\cancel{15x\mp15} \\
\hline
0
\end{array}
$$

$L=(x-1)(2x^2+13x+15)$

$L=(x-1)(2x^2+10x+3x+15)$

$L=(x-1)[2x(x+5)+3(x+5)]$

$L=(x-1)(x+5)(2x+3)$ Ans.

Now $A=(x+5)(x-1)$

$=x^2-x+5x-5$

$\Rightarrow \boxed{A=x^2+4x-5}$

And $B=(x+5)(2x+3)$

$=2x^2+3x+10x+15$

$\Rightarrow \boxed{B=2x^2+13x+15}$ Ans.

Q8: If product of two polynomials is $x^4+6x^3-3x^2-56x-48$ and their L.C.M is $x^3+2x^2-11x-12$. Find their H.C.F.

Solution:

Given $A\times B=x^4+6x^3-3x^2-56x-48$

And $L=x^3+2x^2-11x-12$

We have to find H.C.F = ?

We know $H\times L=A\times B$

$$\Rightarrow H=\frac{A\times B}{L}\rightarrow(1)$$

Putting the values in equation (1)

$$\Rightarrow H=\frac{x^4+6x^3-3x^2-56x-48}{x^3+2x^2-11x-12}$$

$$
\begin{array}{r}
x+4 \\
x^3+2x^2-11x-12\,\big)\;\cancel{x^4}+6x^3-3x^2-56x-48 \\
\cancel{x^4}\pm 2x^3\mp 11x^2\mp 12x \\
\hline
\cancel{4x^3}+\cancel{8x^2}-\cancel{44x}-\cancel{48} \\
\cancel{4x^3}\pm\cancel{8x^2}\mp\cancel{44x}\mp\cancel{48} \\
\hline
0
\end{array}
$$

$\therefore$ Required H.C.F $=\boxed{x+4}$ Ans.

Q9: Waqar wishes to distribute 128 bananas and also 176 apples equally among a certain number of children. Find the highest number of children who can get the fruit in this way.

Solution:

Given total bananas $=128$

And total apples = 176

Let the highest number of children to get both fruits = H.C.F = ?

We find H.C.F of 176 and 128 by division method

$$\begin{array}{r|l} 128 & 176 \quad 1 \\ & 128 \\ \hline 48 & 128 \quad 2 \\ & 96 \\ \hline 32 & 48 \quad 1 \\ & 32 \\ \hline 16 & 32 \quad 2 \\ & 32 \\ \hline & \times \end{array}$$

Hence the highest number of children get fruits = 16 Ans.

Basic Operations on Algebraic Fractions:

An algebraic fraction is the quotient of two algebraic expressions. For example

$$\frac{3x^2+4a}{x+1}$$

EXAMPLE 12

Simplify $\frac{x+y}{3x+2y}+\frac{x-y}{3x+2y}$.

Solution:

$$\frac{x+y}{3x+2y}+\frac{x-y}{3x+2y}$$

$$=\frac{(x+y)+(x-y)}{(3x+2y)} \because L.C.M = 3x+2y$$

$$=\frac{x+\cancel{y}+x-\cancel{y}}{3x+2y}=\frac{2x}{3x+2y}$$

EXAMPLE 13

Simplify $\frac{x-y}{x+y}-\frac{x^2-2y^2}{x^2-y^2}$.

Solution:

$$\frac{x-y}{x+y}-\frac{x^2-2y^2}{x^2-y^2}$$

$$=\frac{x-y}{x+y}-\frac{x^2-2y^2}{(x+y)(x-y)}$$

$$=\frac{(x-y)(x-y)-(x^2-2y^2)}{(x+y)(x-y)}$$

$$=\frac{x^2-2xy+y^2-x^2+2y^2}{(x+y)(x-y)}$$

$$=\frac{3y^2-2xy}{x^2-y^2}$$

EXAMPLE 14

Simplify $\frac{x^2-xy+y^2}{x^3+y^3}+\frac{x^2+xy+y^2}{x^3-y^3}-\frac{1}{x^2-y^2}$.

Solution:

$$\frac{x^2-xy+y^2}{x^3+y^3}+\frac{x^2+xy+y^2}{x^3-y^3}-\frac{1}{x^2-y^2}$$

$$=\frac{\cancel{(x^2-xy+y^2)}}{(x+y)\cancel{(x^2-xy+y^2)}}+\frac{\cancel{(x^2+xy+y^2)}}{(x-y)\cancel{(x^2+xy+y^2)}}$$

$$-\frac{1}{(x+y)(x-y)}$$

$$=\frac{1}{x+y}+\frac{1}{x-y}-\frac{1}{(x+y)(x-y)}$$

$$=\frac{(x-y)+(x+y)-1}{(x+y)(x-y)}$$

$$=\frac{x-\cancel{y}+x+\cancel{y}-1}{x^2-y^2}=\frac{2x-1}{x^2-y^2}$$

EXAMPLE 15

Simplify $\frac{y}{y^2-y-2}-\frac{1}{y^2+5y-14}-\frac{2}{y^2+8y+7}$

Solution:

$$\frac{y}{y^2-y-2}-\frac{1}{y^2+5y-14}-\frac{2}{y^2+8y+7}$$

$$=\frac{y}{(y+1)(y-2)}-\frac{1}{(y-2)(y+7)}-\frac{2}{(y+1)(y+7)}$$

$$=\frac{y(y+7)-(y+1)-2(y-2)}{(y+1)(y-2)(y+7)}$$

$$=\frac{y^2+7y-y-1-2y+4}{(y+1)(y-2)(y+7)}$$

$$=\frac{y^2+4y+3}{(y+1)(y-2)(y+7)}$$

$$=\frac{(y+1)(y+3)}{(y+1)(y-2)(y+7)}=\frac{y+3}{(y-2)(y+7)}$$

EXAMPLE 15

Simplify $\frac{x+4}{x-3}\times\frac{x^2-9}{x^2-x-2}$.

Solution:

$$\frac{x+4}{x-3}\times\frac{x^2-9}{x^2-x-2}$$

$$=\frac{x+4}{x-3}\times\frac{(x+3)(x-3)}{(x-2)(x+1)}\text{ Factorization}$$

Eliminating common factors

$$=\frac{(x+4)(x+3)}{(x-2)(x+1)}$$

EXAMPLE 16

Multiply $\frac{x^2-2x}{2x^2+5x+3}$ **by** $\frac{2x^2-3x-9}{x^2-9}$ **and write the answer in simplified form.**

Solution:

The work may be indicated as follows:

$$\frac{x^2-2x}{2x^2+5x+3}\cdot\frac{2x^2-3x-9}{x^2-9}$$

$$=\frac{x(x-2)(x-3)(2x+3)}{(x+1)(2x+3)(x+3)(x-3)}$$

$$=\frac{x(x-2)}{(x+1)(x+3)}$$

$$=\frac{x^2-2x}{x^2+4x+3}$$

EXERCISE 6.2

Q1: Simplify:

i) $\frac{x}{x+y}+\frac{2y}{x+y}$ ii) $\frac{x+y}{3x+2y}+\frac{x-y}{3x+2y}$

iii) $\frac{3}{y-2}-\frac{2}{y+2}-\frac{y}{y^2-4}$

iv) $\frac{x-y}{x+y}-\frac{x^2-2y^2}{x^2-y^2}$

v) $\frac{x}{2x^2+3xy+y^2}-\frac{x-y}{y^2-4x^2}+\frac{y}{2x^2+xy-y^2}$

vi) $\frac{a}{3x-y}+\frac{a}{3x+y}-\frac{6ax}{9x^2-y^2}$

vii) $\frac{y}{x-y}+\frac{y}{x+y}+\frac{2xy}{x^2+y^2}+\frac{4x^3y}{x^4+y^4}$

viii) $\frac{1}{a^2+7a+10}+\frac{1}{a^2+10a+16}$

ix) $\frac{1}{a-b}+\frac{1}{a+b}+\frac{2a}{a^2+b^2}+\frac{4a^3}{a^4+b^4}$

x) $\frac{x^2-xy+y^2}{x^3+y^3}+\frac{x^2+xy+y^2}{x^3-y^3}-\frac{1}{x^2-y^2}$

Solution:

i) $\frac{x}{x+y}+\frac{2y}{x+y}$

Given $\frac{x}{x+y}+\frac{2y}{x+y}=\frac{x+2y}{x+y}$

(By taking L.C.M)

ii) $\frac{x+y}{3x+2y}+\frac{x-y}{3x+2y}$

By L.C.M

$$=\frac{x+\cancel{y}+x-\cancel{y}}{3x+2y}$$

$$=\frac{2x}{3x+2y}\quad\text{Ans.}$$

iii) $\frac{3}{y-2}-\frac{2}{y+2}-\frac{y}{y^2-4}$

Given $\frac{3}{y-2}-\frac{2}{y+2}-\frac{y}{y^2-4}$

By L.C.M

$$=\frac{3(y+2)-2(y-2)}{(y-2)(y+2)}-\frac{y}{y^2-4}$$

$$=\frac{3y+6-2y+4}{y^2-4}-\frac{y}{y^2-4}$$

$$=\frac{y+10}{y^2-4}-\frac{y}{y^2-4}$$

By L.C.M

$$=\frac{\cancel{y}+10-\cancel{y}}{y^2-4}\Rightarrow\frac{10}{y^2-4}\quad\text{Ans}$$

iv) $\frac{x-y}{x+y}-\frac{x^2-2y^2}{x^2-y^2}$

Given $\frac{x-y}{x+y}-\frac{x^2-2y^2}{x^2-y^2}$

$=\frac{x-y}{x+y}-\frac{x^2-2y^2}{(x+y)(x-y)}$

By L.C.M

$=\frac{(x-y)^2-(x^2-2y^2)}{(x+y)(x-y)}$

$=\frac{(x^2+y^2-2xy)-(x^2-2y^2)}{x^2-y^2}$

$\because x^2-y^2=(x+y)(x-y)$

$=\frac{\cancel{x^2}+y^2-2xy-\cancel{x^2}+2y^2}{x^2-y^2}$

$=\frac{3y^2-2xy}{x^2-y^2}$ Ans.

v) $\frac{x}{2x^2+3xy+y^2}-\frac{x-y}{y^2-4x^2}+\frac{y}{2x^2+xy-y^2}$

Given $\frac{x}{2x^2+3xy+y^2}-\frac{x-y}{y^2-4x^2}+\frac{y}{2x^2+xy-y^2}$

Factorizing the detonator of each fraction

$=\frac{x}{2x^2+2xy+xy+y^2}-\frac{x-y}{(y)^2-(2x)^2}$
$+\frac{y}{2x^2+2xy-xy-y^2}$

$=\frac{x}{2x(x+y)+y(x+y)}\ \frac{x-y}{(y-2x)(y+2x)}$
$+\frac{y}{2x(x+y)-y(x+y)}$

$=\frac{x}{(x+y)(2x+y)}-\frac{x-y}{(y-2x)(2x+y)}$
$+\frac{y}{(x+y)(2x-y)}$

$=\frac{x}{(x+y)(2x+y)}+\frac{(x-y)}{(2x-y)(2x+y)}$
$+\frac{y}{(x+y)(2x-y)}$

$=\frac{x(2x-y)+(x-y)(x+y)+y(2x+y)}{(x+y)(2x+y)(2x-y)}$

$=\frac{2x^2-xy+x^2-\cancel{y^2}+2xy+\cancel{y^2}}{(x+y)\left((2x)^2-(y)^2\right)}$

$=\frac{3x^2+xy}{(x+y)(4x^2-y^2)}$ Ans.

vi) $\frac{a}{3x-y}+\frac{a}{3x+y}-\frac{6ax}{9x^2-y^2}$

Given $\frac{a}{3x-y}+\frac{a}{3x+y}-\frac{6ax}{9x^2-y^2}$

By L.C.M

$=\frac{a(3x+y)+a(3x-y)}{(3x-y)(3x+y)}-\frac{6ax}{9x^2-y^2}$

$=\frac{3ax+\cancel{ay}+3ax-\cancel{ay}}{(3x)^2-(y)^2}-\frac{6ax}{9x^2-y^2}$

$=\frac{\cancel{6ax}}{\cancel{9x^2-y^2}}-\frac{\cancel{6ax}}{\cancel{9x^2-y^2}}$

$=0$ Ans.

vii) $\frac{y}{x-y}+\frac{y}{x+y}+\frac{2xy}{x^2+y^2}+\frac{4x^3y}{x^4+y^4}$

Given $\frac{y}{x-y}+\frac{y}{x+y}+\frac{2xy}{x^2+y^2}+\frac{4x^3y}{x^4+y^4}$

By L.C.M

$=\frac{y(x+y)+y(x-y)}{(x-y)(x+y)}+\frac{2xy}{x^2+y^2}$
$+\frac{4x^3y}{x^4+y^4}$

$=\frac{xy+\cancel{y^2}+xy-\cancel{y^2}}{x^2-y^2}+\frac{2xy}{x^2+y^2}+\frac{4x^3y}{x^4+y^4}$

$=\frac{2xy}{x^2-y^2}+\frac{2xy}{x^2+y^2}+\frac{4x^3y}{x^4+y^4}$

By L.C.M

$=\frac{2xy(x^2+y^2)+2xy(x^2-y^2)}{(x^2-y^2)(x^2+y^2)}+\frac{4x^3y}{x^4+y^4}$

$=\frac{2x^3y+\cancel{2xy^3}+2x^3y-\cancel{2xy^3}}{(x^4-y^4)}+\frac{4x^3y}{x^4+y^4}$

$$=\frac{4x^3y}{x^4-y^4}+\frac{4x^3y}{x^4+y^4}$$

By L.C.M

$$=\frac{4x^3y(x^4+y^4)+4x^3y(x^4-y^4)}{(x^4-y^4)(x^4+y^4)}$$

$$=\frac{4x^7y+\cancel{4x^3y^5}+4x^7y-\cancel{4x^3y^5}}{(x^4)^2-(y^4)^2}$$

$$=\frac{8x^7y}{x^8-y^8} \quad \text{Ans.}$$

viii) $\mathbf{\frac{1}{a^2+7a+10}+\frac{1}{a^2+10a+16}}$

Given $\frac{1}{a^2+7a+10}+\frac{1}{a^2+10a+16}$

By factorizing denominator of each fraction

$$=\frac{1}{a^2+5a+2a+10}+\frac{1}{a^2+2a+8a+16}$$

$$=\frac{1}{a(a+5)+2(a+5)}+\frac{1}{a(a+2)+8(a+2)}$$

By L.C.M

$$=\frac{1}{(a+5)(a+2)}+\frac{1}{(a+2)+(a+8)}$$

$$=\frac{a+8+a+5}{(a+2)(a+5)(a+8)}$$

$$=\frac{2a+13}{(a+2)(a+5)(a+8)} \quad \text{Ans.}$$

ix) $\mathbf{\frac{1}{a-b}+\frac{1}{a+b}+\frac{2a}{a^2+b^2}+\frac{4a^3}{a^4+b^4}}$

Given $\frac{1}{a-b}+\frac{1}{a+b}+\frac{2a}{a^2+b^2}+\frac{4a^3}{a^4+b^4}$

By L.C.M

$$=\frac{a+\cancel{b}+a-\cancel{b}}{(a-b)(a+b)}+\frac{2a}{a^2+b^2}+\frac{4a^3}{a^4+b^4}$$

$$=\frac{2a}{a^2-b^2}+\frac{2a}{a^2+b^2}+\frac{4a^3}{a^4+b^4}$$

By L.C.M

$$=\frac{2a(a^2+b^2)+2a(a^2-b^2)}{(a^2-b^2)(a^2+b^2)}$$

$$+\frac{4a^3}{a^4+b^4}$$

$$=\frac{2a^3+\cancel{2ab^2}+2a^3-\cancel{2ab^2}}{(a^2)^2-(b^2)^2}+\frac{4a^3}{a^4+b^4}$$

$$=\frac{4a^3}{a^4-b^4}+\frac{4a^3}{a^4+b^4}$$

$$=\frac{4a^3(a^4+b^4)+4a^3(a^4-b^4)}{(a^4-b^4)(a^4+b^4)}$$

By L.C.M

$$=\frac{4a^7+\cancel{4a^3b^4}+4a^7-\cancel{4a^3b^4}}{(a^4)^2-(b^4)^2}$$

$$=\frac{8a^7}{a^8-b^8} \quad \text{Ans.}$$

x) $\mathbf{\frac{x^2-xy+y^2}{x^3+y^3}+\frac{x^2+xy+y^2}{x^3-y^3}-\frac{1}{x^2-y^2}}$

Given $\frac{x^2-xy+y^2}{x^3+y^3}+\frac{x^2+xy+y^2}{x^3-y^3}-\frac{1}{x^2-y^2}$

$$=\frac{\cancel{(x^2-xy+y^2)}}{(x+y)\cancel{(x^2-xy+y^2)}}$$

$$+\frac{\cancel{(x^2+xy+y^2)}}{(x-y)\cancel{(x^2+xy+y^2)}}-\frac{1}{(x^2-y^2)}$$

Formula: $x^3-y^3=(x-y)(x^2+xy+y^2)$

$$=\frac{1}{x+y}+\frac{1}{x-y}-\frac{1}{x^2-y^2}$$

By L.C.M

$$=\frac{x-\cancel{y}+x+\cancel{y}}{(x+y)(x-y)}-\frac{1}{x^2-y^2}$$

$$=\frac{2x}{x^2-y^2}-\frac{1}{x^2-y^2}$$

$$=\frac{2x-1}{x^2-y^2} \quad \text{Ans.}$$

Q2: Simplify:

i) $\mathbf{\frac{x^2-25}{5-x}}$

ii) $\mathbf{\frac{x^2+5x+4}{4y^3}\times\frac{2y^2}{x^2+3x+2}}$

iii) $\dfrac{x^2-5x+4}{x^2-3x-4} \div \dfrac{x^3-4x^2+x-4}{2x-1}$

iv) $\dfrac{a(a+b)}{a^3-b^3} \times \dfrac{a^2+ab+b^2}{a^2-b^2}$

v) $\dfrac{7}{x^2-4} \div \dfrac{xy}{x+2}$

vi) $\dfrac{a^3-b^3}{a^4-b^4} \div \dfrac{a^2+ab+b^2}{a^2+b^2}$

vii) $\dfrac{2x}{3x-12} \div \dfrac{x^2-2x}{x^2-6x+8}$

viii) $\dfrac{a^4-8a}{2a^2+5a-3} \times \dfrac{2a-1}{a^2+2a+4} + \dfrac{a^2-2a}{a+3}$

ix) $\dfrac{9-x^2}{x^4+6x^3} \div \dfrac{x^3-2x^2-3x}{x^2+7x+6}$

x) $\dfrac{ax+ab+cx+bc}{a^2-x^2} \times \dfrac{x^2-2ax+a^2}{x^2+(b=a)x+ab}$

Solution:

i) $\dfrac{x^2-25}{5-x}$

Given $\dfrac{x^2-25}{5-x}$

$\Rightarrow \dfrac{(x)^2-(5)^2}{-(x-5)}$

(Take -1 common from denominator)

$= \dfrac{(x+5)\cancel{(x-5)}}{-\cancel{(x-5)}} = \dfrac{x+5}{-1}$

$\Rightarrow -(x+5)$ Ans.

ii) $\dfrac{x^2+5x+4}{4y^3} \times \dfrac{2y^2}{x^2+3x+2}$

Given $\dfrac{x^2+5x+4}{4y^3} \times \dfrac{2y^2}{x^2+3x+2}$

$= \dfrac{x^2+4x+x+4}{{}_2\cancel{4}y \times \cancel{y^2}} \times \dfrac{{}^1\cancel{2}\cancel{y^2}}{x^2+2x+x+2}$

$= \dfrac{x(x+4)+1(x+4)}{2y} \times \dfrac{1}{x(x+2)+1(x+2)}$

$= \dfrac{(x+4)\cancel{(x+1)}}{2y} \times \dfrac{1}{(x+2)\cancel{(x+1)}}$

$= \dfrac{x+4}{2y(x+2)}$ Ans.

iii) $\dfrac{x^2-5x+4}{x^2-3x-4} \div \dfrac{x^3-4x^2+x-4}{2x-1}$

Given $\dfrac{x^2-5x-4}{x^2-3x-4} \div \dfrac{x^3-4x^2+x-4}{2x-1}$

$= \dfrac{x(x-4)-1(x-4)}{x(x-4)+1(x-4)} \times \dfrac{2x-1}{x^2(x-4)+1(x-4)}$

$= \dfrac{\cancel{(x-4)}(x-1)}{(x-4)(x+1)} \times \dfrac{2x-1}{\cancel{(x-4)}(x^2+1)}$

$= \dfrac{(x-1)(2x-1)}{(x+1)(x-4)(x^2+1)}$ Ans.

iv) $\dfrac{a(a+b)}{a^3-b^3} \times \dfrac{a^2+ab+b^2}{a^2-b^2}$

Given $\dfrac{a(a+b)}{a^3-b^3} \times \dfrac{a^2+ab+b^2}{a^2-b^2}$

$= \dfrac{a\cancel{(a+b)}}{(a-b)\cancel{(a^2+ab+b^2)}} \times \dfrac{\cancel{a^2+ab+b^2}}{\cancel{(a+b)}(a-b)}$

$= \dfrac{a}{(a-b)^2}$ Ans.

v) $\dfrac{7}{x^2-4} \div \dfrac{xy}{x+2}$

Given $\dfrac{7}{x^2-4} \div \dfrac{xy}{x+2}$

$\Rightarrow \dfrac{7}{(x+2)(x-2)} \div \dfrac{xy}{x+2}$

$= \dfrac{7}{\cancel{(x+2)}(x-2)} \times \dfrac{\cancel{(x+2)}}{xy}$

$= \dfrac{7}{xy(x-2)}$ Ans.

vi) $\dfrac{a^3-b^3}{a^4-b^4} \div \dfrac{a^2+ab+b^2}{a^2+b^2}$

Given $\dfrac{a^3-b^3}{a^4-b^4} \div \dfrac{a^2+ab+b^2}{a^2+b^2}$

$= \dfrac{(a-b)\cancel{(a^2+ab+b^2)}}{(a^2-b^2)\cancel{(a^2+b^2)}} \times \dfrac{\cancel{(a^2+b^2)}}{\cancel{(a^2+ab+b^2)}}$

$$=\frac{(a-b)}{(a+b)\cancel{(a-b)}}=\frac{1}{a+b} \quad \text{Ans.}$$

vii) $\frac{2x}{3x-12}\div\frac{x^2-2x}{x^2-6x+8}$

Given $\frac{2x}{3x-12}\div\frac{x^2-2x}{x^2-6x+8}$

$$=\frac{2x}{3(x-4)}\div\frac{x(x-2)}{x^2-4x-2x+8}$$

$$=\frac{2x}{3(x-4)}\div\frac{x(x-2)}{x(x-4)-2(x-4)}$$

$$=\frac{2x}{3(x-4)}\div\frac{x\cancel{(x-2)}}{(x-4)\cancel{(x-2)}}$$

$$\Rightarrow\frac{2\cancel{x}}{3\cancel{(x-4)}}\times\frac{\cancel{(x-4)}}{\cancel{x}}=\frac{2}{3} \quad \text{Ans.}$$

viii) $\frac{a^4-8a}{2a^2+5a-3}\times\frac{2a-1}{a^2+2a+4}+\frac{a^2-2a}{a+3}$

Given $\frac{a^4-8a}{2a^2+5a-3}\times\frac{2a-1}{a^2+2a+4}+\frac{a^2-2a}{a+3}$

$$=\frac{a(a^3-8)}{2a^2+6a-a-3}\times\frac{2a-1}{a^2+2a+4}\div\frac{a(a-2)}{a+3}$$

$$=\frac{a(a-2)(a^2+2a+4)}{2a(a+3)-1(a+3)}\times\frac{2a-1}{a^2+2a+4}\div\frac{a(a-2)}{(a+3)}$$

$$=\frac{a\cancel{(a-2)}\cancel{(a^2+2a+4)}}{\cancel{(a+3)}\cancel{(2a-1)}}\times\frac{\cancel{2a-1}}{\cancel{a^2+2a+4}}\times\frac{\cancel{(a+3)}}{a\cancel{(a-2)}}=1 \quad \text{Ans.}$$

ix) $\frac{9-x^2}{x^4+6x^3}\div\frac{x^3-2x^2-3x}{x^2+7x+6}$

Given $\frac{9-x^2}{x^4+6x^3}\div\frac{x^3-2x^2-3x}{x^2+7x+6}$

$$=\frac{(3)^2-(x)^2}{x^3(x+6)}\div\frac{x(x^2-2x-3)}{x^2+7x+6}$$

$$=\frac{(3+x)(3-x)}{x^3(x+6)}\div\frac{x(x^2-3x+x-3)}{x^2+6x+x+6}$$

$$=\frac{(3-x)(3+x)}{x^3(x+6)}\div\frac{x[x(x-3)+1(x-3)]}{x(x+6)+1(x+6)}$$

$$=\frac{-\cancel{(x-3)}(x+3)}{x^3\cancel{(x+6)}}\times\frac{\cancel{(x+6)}\cancel{(x+1)}}{x\cancel{(x-3)}\cancel{(x+1)}}$$

$$=\frac{-(x+3)}{x^4} \quad \text{Ans.}$$

x) $\frac{ax+ab+cx+bc}{a^2-x^2}\times\frac{x^2-2ax+a^2}{x^2+(b+a)x+ab}$

Given $\frac{ax+ab+cx+bc}{a^2-x^2}\times\frac{x^2-2ax+a^2}{x^2+(b+a)x+ab}$

$$=\frac{ax+cx+ab+bc}{(a-x)(a+x)}\times\frac{(x)^2-2(a)(x)+(a)^2}{x^2+bx+ax+ab}$$

$$=\frac{x(a+c)+b(a+c)}{(a-x)(a+x)}\times\frac{(x-a)^2}{x(a+b)+a(x+b)}$$

$$=\frac{(a+c)\cancel{(x+b)}}{\cancel{(a-x)}(a+x)}\times\frac{\cancel{(a-x)}(a-x)}{\cancel{(x+b)}(x+a)}$$

$$=\frac{(a+c)(a-x)}{(a-x)(a+x)}$$

$$=\frac{(a+c)(a-x)}{(a+x)^2} \quad \text{Ans.}$$

Square Root of Algebraic Expressions:

Square root of algebraic expression can be found by two methods:

1. Square root by factorization
2. Square root by division

EXAMPLE 20

Find the square root if $x^2+ax+\frac{1}{4}a^2$ by factorization.

Solution:

We are to find $\sqrt{x^2+ax+\frac{1}{4}a^2}$

$$x^2+ax+\frac{1}{4}a^2=(x)^2+2.\frac{1}{2}a.x+\left(\frac{1}{2}a\right)^2$$

$$=\left(x+\frac{1}{2}a\right)^2$$

$$\therefore \pm\sqrt{x^2+ax+\frac{1}{4}a^2}=x+\frac{1}{2}a \qquad \text{Ans.}$$

EXAMPLE 21

Find square root of $x^2+\frac{1}{x^2}-10\left(x+\frac{1}{x}\right)+27$

Solution:

Given $x^2+\frac{1}{x^2}-10\left(x+\frac{1}{x}\right)+27$

$$=x^2+\frac{1}{x^2}-10\left(x+\frac{1}{x}\right)+25+2$$

$$=x^2+\frac{1}{x^2}+2-10\left(x+\frac{1}{x}\right)+25$$

$$=\left(x+\frac{1}{x}\right)^2-2\times5\left(x+\frac{1}{x}\right)+(5)^2$$

$$=\left(x+\frac{1}{x}\right)^2-2\left(x+\frac{1}{x}\right).5+(5)^2$$

$$=\left(x+\frac{1}{x}-5\right)^2 \quad \because a^2-2ab+b^2=(a-b)^2$$

Therefore $\pm\sqrt{x^2+\frac{1}{x^2}-10\left(x+\frac{1}{x}\right)+27}$

$$=\sqrt{\left(x+\frac{1}{x}-5\right)^2}=\left(x-5+\frac{1}{x}\right) \text{ Ans.}$$

EXAMPLE 22

Find the square root of $16x^4-24x^3+25x^2-12x+4$.

Solution:

$\sqrt{16x^4}=4x^2$

2nd element of square root

$$=\frac{-24x^3}{8x^2}=-3x$$

3rd element of square root

$$=\frac{16x^2}{8x^2}=2$$

$$\begin{array}{r|l}
 & 4x^2-3x+2 \\
\hline
4x^2 & 16x^4-24x^3+25x^2-12x+4 \\
 & \pm16x^4 \\
\hline
8x^2-3x & -24x^3+25x^2-12x+4 \\
 & \mp24x^3\pm9x^2 \\
\hline
8x^2-6x+2 & +16x^2-12x+4 \\
 & \underline{\pm16x^2\mp12x\pm4} \\
 & 0
\end{array}$$

Double of the first elements of square root

Double of the first two elements of square root

Therefor $\sqrt{16x^4-24x^3+25x^2-12x+4}$

$$=\pm(4x^2-3x+2) \text{ Ans.}$$

Rules for finding square root by division method:

1. Write the expression in descending order.
2. Take square root of first element which is $4x^2$.
3. Subtract the square of $4x^2$ from given expression and get remainder $-24x^3+25x^2-12x+4$.
4. Find the 2nd element of square root by dividing the first term of remainder.

EXERCISE 6.3

Q1: Find the square root by factorization method.

i) x^2+4x+4

ii) $(x+y)^2+6(x+y)+9$

iii) $x^2y^2-8xy+16$

iv) $x^2+\frac{1}{x^2}-8\left(x+\frac{1}{x}\right)^2+18$

v) $x(x+1)(x+2)(x+3)+1$

vi) $\left(x+\frac{1}{x^2}\right)^2-2\left(x-\frac{1}{x}\right)^2+\frac{9}{4}$

vii) $\left(x^2+\frac{1}{x^2}\right)^2-4\left(x+\frac{1}{x}\right)^2+12$

viii) $\dfrac{4x^6-12x^3y^3+9y^6}{9x^4+24x^2y^2+16y^4}$

Solution:

i) x^2+4x+4

Formula; $a^2+2ab+b^2=(a+b)^2$

Given x^2+4x+4

$=(x)^2+2(x)(2)+(2)^2$

$=(x+2)^2$

Taking square root of both sides

$\sqrt{x^2+4x+4}=\sqrt{(x+2)^2}=\pm(x+2)$

$\sqrt{x^2+4x+4}=\pm(x+2)$ Ans.

ii) $(x+y)^2+6(x+y)+9$

Given $(x+y)^2+6(x+y)+9$

$=(x+2)^2+2(x+y)(3)+(3)^2$

$=(x+y+3)^2$

Taking square root of both sides

$\sqrt{(x+y)^2+6(x+y)+9}$

$=\sqrt{(x+y+3)^2}=\pm(x+y+3)$ Ans.

iii) $x^2y^2-8xy+16$

Given $x^2y^2-8xy+16$

$=(xy)^2-2(xy)(4)+(4)^2$

$=(xy-4)^2$

Taking square root of both sides

$\sqrt{x^2y^2-8xy+16}$

$=\sqrt{(xy-4)^2}$

$=\pm(xy-4)$ Ans.

iv) $x^2+\dfrac{1}{x^2}-8\left(x+\dfrac{1}{x}\right)^2+18$

Given $x^2+\dfrac{1}{x^2}-8\left(x+\dfrac{1}{x}\right)^2+18$

$=x^2+\dfrac{1}{x^2}-8\left(x+\dfrac{1}{x}\right)+16+2$

$=x^2+\dfrac{1}{x^2}+2-8\left(x+\dfrac{1}{x}\right)+16$

$=\left(x+\dfrac{1}{x}\right)^2-2\left(x+\dfrac{1}{x}\right)(4)+(4)^2$

$=\left(x+\dfrac{1}{x}-4\right)^2$

Taking square root of both sides

$\sqrt{x^2+\dfrac{1}{x^2}-8\left(x+\dfrac{1}{x}\right)+18}$

$=\sqrt{\left(x-4+\dfrac{1}{x}\right)^2}$

$=\pm\left(x-4+\dfrac{1}{x}\right)$ Ans.

v) $x(x+1)(x+2)(x+3)+1$

Given $x(x+1)(x+2)(x+3)+1$

$=x(x+3)(x+1)(x+2)+1$

$=(x^2+3x)(x^2+2x+x+2)+1$

$=\left(x^2+3x\right)\left(x^2+3x+2\right)+1$

Let $x^2+3x=y \Rightarrow y(y+2)+1$

$\Rightarrow y^2+2y+1 \Rightarrow (y+1)^2$

Taking square root of both sides

$\sqrt{x(x+1)(x+2)(x+3)+1}$

$=\sqrt{(y+1)^2}=\pm(y+1)$

Put $y=x^2+3x$

$=\pm(x^2+3x+1)$ Ans.

vi) $\left(x^2+\dfrac{1}{x^2}\right)^2-2\left(x-\dfrac{1}{x}\right)^2+\dfrac{9}{4}$

Given $\left(x^2+\dfrac{1}{x^2}\right)^2-2\left(x^2-\dfrac{1}{x}\right)^2+\dfrac{9}{4}$

$=\left(x^2+\dfrac{1}{x^2}\right)^2-2\left(x^2+\dfrac{1}{x^2}-2\right)+\dfrac{9}{4}$

$=\left(x^2+\dfrac{1}{x^2}\right)^2-2\left(x^2+\dfrac{1}{x^2}\right)+4+\dfrac{9}{4}$

$=\left(x^2+\dfrac{1}{x^2}\right)^2-2\left(x^2+\dfrac{1}{x^2}\right)+\dfrac{25}{4}$

$=\left(x^2+\dfrac{1}{x^2}\right)^2-2\left(x^2+\dfrac{1}{x^2}\right)\left(\dfrac{5}{2}\right)+\left(\dfrac{5}{2}\right)^2$

$=\left(x^2+\dfrac{1}{x^2}-\dfrac{5}{2}\right)^2$

Taking square root of both sides

$$\sqrt{\left(x^2+\frac{1}{x^2}\right)^2-2\left(x-\frac{1}{x}\right)^2+\frac{9}{4}}$$

$$=\sqrt{\left(x^2-\frac{5}{2}+\frac{1}{x^2}\right)^2}$$

$$=\pm\left(x^2-\frac{5}{2}+\frac{1}{x^2}\right) \text{ Ans.}$$

vii) $\left(x^2+\frac{1}{x^2}\right)^2-4\left(x+\frac{1}{x}\right)^2+12$

Given $\left(x^2+\frac{1}{x^2}\right)^2-4\left(x+\frac{1}{x}\right)^2+12$

$$=\left(x^2+\frac{1}{x^2}\right)^2-4\left(x^2+\frac{1}{x^2}+2\right)+12$$

$$=\left(x^2+\frac{1}{x^2}\right)^2-4\left(x^2+\frac{1}{x^2}\right)-8+12$$

$$=\left(x^2+\frac{1}{x^2}\right)^2-4\left(x^2+\frac{1}{x^2}\right)+4$$

$$=\left(x^2+\frac{1}{x^2}\right)^2-2\left(x^2+\frac{1}{x^2}\right)(2)+(2)^2$$

$$=\left(x^2+\frac{1}{x^2}-2\right)^2$$

Taking square root of both sides

$$\sqrt{\left(x^2+\frac{1}{x^2}\right)^2-4\left(x+\frac{1}{x}\right)^2+12}$$

$$=\sqrt{\left(x^2-2+\frac{1}{x^2}\right)^2}$$

$$\Rightarrow \pm\left(x^2-2+\frac{1}{x^2}\right) \text{ Ans.}$$

viii) $\dfrac{4x^6-12x^3y^3+9y^6}{9x^4+24x^2y^2+16y^4}$

Given $\dfrac{4x^6-12x^3y^3+9y^6}{9x^4+24x^2y^2+16y^4}$

$$=\frac{(2x^3)-2(2x^3)(3y^3)+(3y^3)^2}{(3x^2)^2+2(3x^2)(4y^2)+(4y^2)^2}$$

$$=\frac{(2x^3+3y^3)^2}{(3x^2+4y^2)^2}$$

$$=\left(\frac{2x^3+3y^3}{3x^2+4y^2}\right)^2$$

Taking square root of both sides

$$\sqrt{\frac{4x^6-12x^3y^3+9y^6}{9x^4+24x^2y^2+16y^4}}$$

$$=\pm\sqrt{\left(\frac{2x^3+3y^3}{3x^2+4y^2}\right)^2}$$

$$=\pm\left(\frac{2x^3+3y^3}{3x^2+4y^2}\right) \text{ Ans.}$$

Q2: Find the square root of the following by division method:

i) $4x^4-4x^3+13x^2-6x+9$

ii) $x^4+x^3-\frac{31}{4}x^2-4x+16$

iii) $x^2-2x+1+2xy-2y+y^2$

iv) $\left(x^2-\frac{1}{x^2}\right)^2-12\left(x^2-\frac{1}{x^2}\right)+36$

Solution:

i) $4x^4-4x^3+13x^2-6x+9$

Given $4x^4-4x^3+13x^2-6x+9$

	$2x^2-x+3$
$2x^2$	$\cancel{4x^4}-4x^3+13x^2-6x+9$
	$\pm\cancel{4x^4}$
$4x^2-x$	$-\cancel{4x^3}+13x^2$
	$\mp\cancel{4x^3}\pm x^2$
$4x^2-2x+3$	$\cancel{12x^2}-\cancel{6x}+\cancel{9}$
	$\cancel{12x^2}\mp\cancel{6x}\pm\cancel{9}$
	0

Hence square root $=\boxed{\left(2x^2-x+3\right)}$ Ans.

ii) $x^4+x^3-\frac{31}{4}x^2-4x+16$

Given $x^4+x^3-\frac{31}{4}x^2-4x+16$

$$
\begin{array}{r|l}
 & x^2+\frac{1}{2}x-4 \\
\hline
x^2 & \cancel{x^4}+x^3-\frac{31}{4}x^2-4x+16 \\
 & \pm\cancel{x^4} \\
\hline
2x^2+\frac{1}{2}x & \cancel{x^3}-\frac{31}{4}x^2 \\
 & \pm\cancel{x^3}\pm\frac{1}{4}x^2 \\
\hline
2x^2+x-4 & -\cancel{8x^2}-\cancel{4x}+\cancel{16} \\
 & \mp\cancel{8x^2}\mp\cancel{4x}\pm\cancel{16} \\
\hline
 & 0
\end{array}
$$

Hence square root $=\left(x^2+\frac{1}{2}x-4\right)$ Ans.

iii) $x^2-2x+1+2xy-2y+y^2$

Given $x^2-2x+1+2xy-2y+y^2$

$$
\begin{array}{r|l}
 & x-1+y \\
\hline
x & \cancel{x^2}-2x+1+2xy-2y+y^2 \\
 & \cancel{x^2} \\
\hline
2x-1 & -\cancel{2x}+\cancel{1} \\
 & \mp\cancel{2x}\pm\cancel{1} \\
\hline
2x-2+y & \cancel{2xy}-\cancel{2y}+\cancel{y^2} \\
 & \cancel{2xy}\mp\cancel{2y}\pm\cancel{y^2} \\
\hline
 & 0
\end{array}
$$

Hence square root $=(x-1+y)$ Ans.

iv) $\left(x^2-\frac{1}{x^2}\right)^2-12\left(x^2-\frac{1}{x^2}\right)+36$

Given $\left(x^2-\frac{1}{x^2}\right)^2-12\left(x^2-\frac{1}{x^2}\right)+36$

$=(x^2)^2+\left(\frac{1}{x^2}\right)^2-2(\cancel{x^2})\left(\frac{1}{\cancel{x^2}}\right)-12x^2+\frac{12}{x^2}+36$

$=x^4+\frac{1}{x^4}-2-12x^2+\frac{12}{x^2}+36$

Write in descending order

$=x^4-12x^2+34+\frac{12}{x^2}+\frac{1}{x^4}$

$$
\begin{array}{r|l}
 & x^2-6-\frac{1}{x^2} \\
\hline
x^2 & \cancel{x^4}-12x^2+34+\frac{12}{x^2}+\frac{1}{x^4} \\
 & \pm\cancel{x^4} \\
\hline
2x^2-6 & -\cancel{12x^2}+34 \\
 & \mp\cancel{12x^2}\pm36 \\
\hline
2x^2-12-\frac{1}{x^2} & -\cancel{2}+\cancel{\frac{12}{x^2}}+\cancel{\frac{1}{x^4}} \\
 & \mp\cancel{2}\pm\cancel{\frac{12}{x^2}}\pm\cancel{\frac{1}{x^4}} \\
\hline
 & 0
\end{array}
$$

$\therefore$ Required square root $=\left(x^2-6-\frac{1}{x^2}\right)$ Ans.

Q3: For what value of k the expression $4x^4+32x^2+96+\frac{128}{x^2}+\frac{k}{x^4}$ become a perfect square.

Solution:

Given $4x^4+32x^2+96+\frac{128}{x^2}+\frac{k}{x^4}$

$$
\begin{array}{r|l}
 & 2x^2+8+\frac{8}{x^2} \\
\hline
2x^2 & \cancel{4x^4}+32x^2+96+\frac{128}{x^2}+\frac{k}{x^4} \\
 & \pm\cancel{4x^4} \\
\hline
4x^2+8 & \cancel{32x^2}+96 \\
 & \pm\cancel{32x^2}\pm64 \\
\hline
\end{array}
$$

$$
\begin{array}{r|l}
4x^2+16+\dfrac{8}{x^2} & 32+\dfrac{128}{x^2}+\dfrac{k}{x^4} \\
 & \pm 32\pm\dfrac{128}{x^2}\pm\dfrac{64}{x^4} \\
\hline
 & \dfrac{k}{x^4}-\dfrac{64}{x^4}
\end{array}
$$

The given expression will be a perfect square if $\dfrac{k}{x^4}-\dfrac{64}{x^4}=0$

$\Rightarrow \dfrac{k}{x^4}=\dfrac{64}{x^4} \Rightarrow \boxed{k=64}$

i) What should be added to?
ii) What should be subtracted from?
iii) For what value of x the expression $4x^4-12x^3+17x^2-13x+6$ so that it becomes a perfect square?

Solution:

$$
\begin{array}{r|l}
2x^2 & \cancel{4x^4}-12x^3+17x^2-13x+6 \\
 & \cancel{4x^4} \\
\hline
4x^2-3x & -\cancel{12x^3}+17x^2 \\
 & \mp\cancel{12x^3}\pm 9x^2 \\
\hline
4x^2-6x+2 & \cancel{8x^2}-13x+6 \\
 & \pm\cancel{8x^2}\mp 12x\pm 4 \\
\hline
 & -x+2
\end{array}
$$

i) If we add $x-2$ then the given expression will be a perfect square.
ii) If we subtract $-x+2$ from the given expression then it will be a perfect square.
iii) If the expression is a perfect square then $-x+2=0 \Rightarrow -x=-2$
$\Rightarrow \boxed{x=2}$.

Q4: What should be subtracted and added to the expression $x^4-4x^3+10x+7$, so that the expression is made perfect square?

Given: $x^4-4x^3+10x+7$

Required: To find what should be subtracted and added to the expression that is made perfect square.

Solution:

As $x^4-4x^3+10x+7$

$$
\begin{array}{r|l}
 & x^2-2x-2 \\
\hline
x^2 & \cancel{x^4}-4x^3+10x+7 \\
 & \pm\cancel{x^4} \\
\hline
2x^2-2x & -\cancel{4x^3}+10x+7 \\
 & \mp\cancel{4x^3}\pm 4x^2 \\
\hline
2x^2-4x-2 & \cancel{-4x^2}+10x+7 \\
 & \mp\cancel{4x^2}\pm 8x\pm 4 \\
\hline
 & 2x+3
\end{array}
$$

So $-(2x+3)$ should be added or subtracted $(2x+3)$ to make the expression a perfect square.

Result: Hence $-(2x+3)$ should be added or $(2x+3)$ should be subtracted to made the expression a perfect square.

Q5: Find the value of l & m for which the following expressions will become perfect squares.

i) $x^4+4x^3+16x^2+\ell x+m$
ii) $49x^4-70x^3+109x^2+\ell x-m$

Solution:

i) $x^4+4x^3+16x^2+\ell x+m$

Given $x^4+4x^3+16x^2+\ell x+m$

$$
\begin{array}{r|l}
 & x^2+2x+6 \\
\hline
x^2 & \cancel{x^4}+4x^3+16x^2+lx+m \\
 & \pm\cancel{x^4} \\
\hline
2x^2+2x & \cancel{4x^3}+16x^2 \\
 & \pm\cancel{4x^3}\pm 4x^2 \\
\hline
2x^2+4x+6 & \cancel{12x^2}+lx+m \\
 & \pm\cancel{12x^2}\pm 24x\pm 36 \\
\hline
 & lx-24x+m-36
\end{array}
$$

The given expression will be a perfect square, if $lx-24x=0$

$\Rightarrow l\cancel{x}=24\cancel{x} \Rightarrow l=24$

And $m-36=0 \Rightarrow m=36$

$\boxed{\therefore \ell=24 \text{ and } m=36}$ Ans.

ii) $49x^4 - 70x^3 + 109x^2 + lx - m$

Given $49x^4 - 70x^3 + 109x^2 + lx - m$

	$7x^2 - 5x + 6$
$7x^2$	$\cancel{49x^4} - 70x^3 + 109x^2 + lx - m$
	$\pm\cancel{49x^4}$
$14x^2 - 5x$	$-\cancel{70x^3} + 109x^2$
	$\mp\cancel{70x^3} \pm 25x^2$
$14x^2 - 10x + 6$	$\cancel{84x^2} + lx - m$
	$\pm\cancel{84x^2} \mp 60x \pm 36$
	$lx + 60x - m - 36$

For perfect square $lx + 60x = 0$

$\Rightarrow l = -60$

And $-m - 36 = 0$

$\Rightarrow m = -36$ Ans.

Review Exercise 6

Q1: Select the correct answer.

i) H.C.F of $a^3 - 8b^3$ and $a^2 - 4ab + 4b^2$ is……….

✓**(a)** $a - 2b$

(b) $a^2 + 2ab + b^2$

(c) $a + 2b$

(d) $(a + 2b)^2$

ii) L.C.M of $(2x + 3y)^5$ and $(2x + 3y)^3$ is:

(a) $2x + 3y$

(b) $(2x + 3y)^3$

(c) $(2x + 3y)^2$

✓**(d)** $(2x + 3y)^5$

iii) H.C.F of $a^3 - b^3$ and $a^2 + ab + b^2$ is:

(a) $a + b$

✓**(b)** $a^2 + ab + b^2$

(c) $a - b$

(d) $(a - b)^2$

iv) L.C.M of $(a - b)^4$ and $(a - b)^3$ is:

(a) $(a - b)$

(b) $(a - b)^3$

✓(c) $(a - b)^4$

(d) $(a - b)^7$

v) Reduce to lowest terms $\frac{10(x+3)(x-2)}{15(x-2)}$

✓(a) $\frac{2(x+3)}{3}$

(b) $2x$

(c) $\frac{10(x+3)}{15}$

(d) $2(x+1)$

vi) Simplified form of $\frac{b}{25a^2 - b^2} - \frac{1}{5a - b}$ is…….

(a) $\frac{5a}{25a^2 - b^2}$

(b) $\frac{+5a}{5a-b}$

(c) $\frac{-5a}{5a+b}$

✓(d) $\frac{-5a}{25a^2-b^2}$

vii) $\frac{5}{x^2-x-2}+\frac{3}{x^2+4x+3}=$

(a) $\frac{8x+21}{(x-1)(x+2)(x+3)}$

(b) $\frac{8x-3}{(x+1)(x-2)(x+3)}$

(c) $\frac{8x+6}{(x-1)(x+2)(x+3)}$

✓(d) $\frac{8x+9}{(x-1)(x+2)(x+3)}$

viii) $\frac{x^2-2x-3}{3x^2+x-2}=$ ....

✓(a) $\frac{x-3}{3x-2}$

(b) $\frac{-2x-3}{3x-2}$

(c) $\frac{-2x-2}{x+1}$

(d) $\frac{x-3}{3x+2}$

ix) L.C.M = ...

(a) $\frac{H.C.F}{A\times B}$

✓(b) $\frac{A\times B}{H.C.F}$

(c) $\frac{A}{H.C.F}$

(d) $\frac{B}{H.C.F}$

x) L.C.M of a^2-a+1 and a^3+1 is:

(a) $a+1$

(b) a^2-a+1

✓(c) a^3+1

(d) a^2+a+1

Q2: Simplify the following:

i) $\frac{5}{2s+4}-\frac{3}{s^2+3s+2}+\frac{s}{s^2-s-2}$

ii) $\frac{a}{(c-a)(a-b)}+\frac{b}{(a-b)(b-c)}+\frac{c}{(b-c)(c-a)}$

iii) $\frac{x^2-4}{xy^2}\cdot\frac{2xy}{x^2-4x+4}$

iv) $\frac{a^3-b^3}{a^4-b^4}\div\frac{a^2+ab+b^2}{a^2+b^2}$

Solution:

i) $\frac{5}{2s+4}-\frac{3}{s^2+3s+2}+\frac{s}{s^2-s-2}$

Given $\frac{5}{2s+4}-\frac{3}{s^2+3s+2}+\frac{s}{s^2-s-2}$

$=\frac{5}{2(s+2)}-\frac{3}{s^2+2s+s+2}$

$+\frac{s}{s^2-2s+s-2}$

$=\frac{5}{2(s+2)}-\frac{3}{s(s+2)+1(s+2)}$

$+\frac{s}{s(s-2)+1(s-2)}$

$=\frac{5(s-2)(s+1)-3\times 2(s-2)+2s(s+2)}{2(s+2)(s-2)(s+1)}$

$=\frac{5(s^2-s-2)-6(s-2)+2s(s+2)}{2(s+2)(s-2)(s+1)}$

$=\frac{5s^2-5s-10-6s+12+2s^2+4s}{2(s+2)(s-2)(s+1)}$

$=\frac{7s^2-7s+2}{2(s^2-4)(s+1)}$ Ans.

ii) $\frac{a}{(c-a)(a-b)}+\frac{b}{(a-b)(b-c)}+\frac{c}{(b-c)(c-a)}$

Given $\frac{a}{(c-a)(a-b)}+\frac{b}{(a-b)(b-c)}+\frac{c}{(b-c)(c-a)}$

Taking L.C.M

$$=\frac{a(b-c)+b(c-a)+c(a-b)}{(c-a)(a-b)(b-c)}$$

$$=\frac{\cancel{ab}-\cancel{ac}+\cancel{bc}-\cancel{ab}+\cancel{ac}-\cancel{bc}}{(a-b)(b-c)(c-a)}$$

$$=\frac{0}{(a-b)(c-a)(b-c)}$$

$=0$ Ans.

iii) $\dfrac{x^2-4}{xy^2}\cdot\dfrac{2xy}{x^2-4x+4}$

Given $\dfrac{x^2-4}{xy^2}\cdot\dfrac{2xy}{x^2-4x+4}$

Formula; $a^2-b^2=(a-b)(a+b)$

$$=\frac{(x)^2-(2)^2}{xy^2}\times\frac{2xy}{(x)^2-2(x)(2)+(2)^2}$$

$$=\frac{(x+2)\cancel{(x-2)}}{\cancel{x}y^{\cancel{2}}}\times\frac{2\cancel{x}\cancel{y}}{(x-2)^{\cancel{2}}}$$

$$=\frac{2(x+2)}{y(x-2)}$$ Ans.

iv) $\dfrac{a^3-b^3}{a^4-b^4}\div\dfrac{a^2+ab+b^2}{a^2+b^2}$

Given $\dfrac{a^3-b^3}{a^4-b^4}\div\dfrac{a^2+ab+b^2}{a^2+b^2}$

$$=\frac{(a)^3-(b)^3}{(a^2)^2-(b^2)^2}\div\frac{a^2+ab+b^2}{a^2+b^2}$$

$$=\frac{(a-b)(a^2+ab+b^2)}{(a^2-b^2)(a^2+b^2)}\div\frac{a^2+ab+b^2}{a^2+b^2}$$

$$=\frac{\cancel{(a-b)}\cancel{(a^2+ab+b^2)}}{\cancel{(a-b)}(a+b)\cancel{(a^2+b^2)}}\times\frac{\cancel{a^2+b^2}}{\cancel{(a^2+ab+b^2)}}=\frac{1}{a+b}$$ Ans.

Q3: Find L.C.M of $x^3-6x^2+11x-6$ and x^3-4x+3.

Solution:

Given $x^3-6x^2+11x-6$ and x^3-4x+3

As $x^3-6x^2+11x-6$

Since by using factor theorem, we can factorize

$p(x)=x^3-6x^2+11x-6$

Put $x=1$

$\Rightarrow p(1)=(1)^3-6(1)^2+11(1)-6$

$\Rightarrow p(1)=1-6+11-6$

$\Rightarrow p(1)=12-12$

$\Rightarrow p(1)=0$

Hence $x=1$ or $(x-1)$ we can find other factors by division, so

$$\frac{x^3-6x^2+11x-6}{(x-1)}$$

$$\begin{array}{r|l}
 & x^2-5x+6 \\ \hline
x-1 & \cancel{x^3}-6x^2+11x-6 \\
 & \pm\cancel{x^3}\mp x^2 \\ \hline
 & -\cancel{5x^2}+11x-6 \\
 & \pm\cancel{5x^2}\pm 5x \\ \hline
 & \cancel{6x}-\cancel{6} \\
 & \pm\cancel{6x}\mp\cancel{6} \\ \hline
 & 0
\end{array}$$

$\Rightarrow p(x)=(x-1)(x^2-5x+6)$

Let $Q(x)=x^3-4x+3$

$\Rightarrow Q(1)=(1)^3-4(1)+3$

$\Rightarrow Q(1)=1-4+3$

$\Rightarrow Q(1)=4-4=0$

$\Rightarrow x=1$ or $(x-1)$ is a factor of x^3-4x+3

Simple division

$$\begin{array}{r|l}
 & x^2+x-3 \\ \hline
x-1 & \cancel{x^3}-4x+3 \\
 & \pm\cancel{x^3}\mp x^2 \\ \hline
 & \cancel{x^2}-4x+3 \\
 & \pm\cancel{x^2}\mp x \\ \hline
 & -\cancel{3x}+\cancel{3} \\
 & \mp\cancel{3x}\pm\cancel{3} \\ \hline
 & 0
\end{array}$$

$\Rightarrow Q(x)=(x-1)(x^2+x-3)$

L.C.M of

$p(x)$ and $Q(x)$ is (x^2-5x+6) and (x^3-4x+3)

OR

$L=(x^2-5x+6)(x-1)(x^2+x-3)$ Ans.

Q4: Find the square root of:

i) $4x^2-12x+9$

ii) $x^4+4x^2+6x^2+4x+1$

Solution:

i) $4x^2-12x+9$

Given $4x^2-12x+9$

As $4x^2-12x+9$

$$=4x^2-6x-6x+9$$
$$=2x(2x-3)-3(2x-3)$$
$$=(2x-3)(2x-3)$$
$$=(2x-3)^2 \quad \because (a-b)^2=(a-b)(a-b)$$

Taking square root

$\sqrt{(2x-3)^2}=2x-3$

Hence $4x^2-12x+9=2x-3$ Ans.

ii) $x^4+4x^2+6x^2+4x+1$

Given $x^4+4x^3+6x^2+4x+1$

As $x^4+4x^3+6x^2+4x+1$

Now we find square root

$$\begin{array}{r|l} & x^2+2x+1 \\ \hline x^2 & \cancel{x^4}+4x^3+6x^2+4x+1 \\ & \pm\cancel{x^4} \\ \hline 2x^2+2x & \cancel{4x^3}+6x^2+4x+1 \\ & \pm4x^3\pm4x^2 \\ \hline & \cancel{2x^2}+\cancel{4x}+\cancel{1} \\ & \pm\cancel{2x^2}\pm\cancel{4x}\pm\cancel{1} \\ \hline & 2x+3 \end{array}$$

So $\sqrt{x^4+4x^3+6x^2+4x+1}$

$=x^2+2x+1$

Hence square root $=x^2+2x+1$ Ans.

Think:

Q5: Simplify:

$$\frac{x^3-y^3}{x^3+z^3}\times\frac{x^2+xy+xz+yz}{x^4+x^2y^2+y^4}\times\frac{x^3+y^3}{x^2-y^2}$$

Solution:

Given $\dfrac{x^3-y^3}{x^3+z^3}\times\dfrac{x^2+xy+xz+yz}{x^4+x^2y^2+y^4}\times\dfrac{x^3+y^3}{x^2-y^2}$

As $\dfrac{x^3-y^3}{x^3+z^3}\times\dfrac{x^2+xy+xz+yz}{x^4+x^2y^2+y^4}\times\dfrac{x^3+y^3}{x^2-y^2}$

$$=\frac{(x-y)(x^2+xy+y^2)}{(x+z)(x^2+xz+z^2)}\times\frac{x(x+y)+z(x+y)}{x^4+x^2y^2+y^4}$$
$$\times\frac{(x+y)(x^2-xy+y^2)}{(x-y)(x+y)}$$
$$=\frac{(x-y)(x^2+xy+y^2)(x+z)(x+y)(x+y)(x^2-xy+y^2)}{(x+z)(x^2+xz+z^2)(x^4+x^2y^2+y^4)(x-y)(x+y)}$$
$$=\frac{(x^2+xy+y^2)(x^2-xy+y^2)(x+y)}{(x^2+xz+z^2)(x^4+x^2y^2+y^4)}$$
$$=\frac{(x^4+x^2y^2+y^4)(x+y)}{(x^4+x^2y^2+y^4)(x^2+xz+z^2)}$$
$$=\frac{x+y}{(x^2+xz+z^2)}$$

Hence the simplified form is

$$=\frac{x+y}{(x^2+xz+z^2)} \quad \text{Ans.}$$

***

Additional MCQs of Unit 6:
Algebraic Manipulation

1. The highest number of factors common to all given expression is called..........
   (a) L.C.M (b) H.C.F (c) Remainder (d) Factor
   ✓ **Ans. (b) H.C.F**

2. To find H.C.F there are..........methods.
   (a) One (b) Three (c) Two (d) Four
   ✓ **Ans. (c) Two**

3. The H.C.F of $x^2 - y^2$ and $x^2 - xy$ is..........
   (a) $x+y$ (b) $x-y$ (c) x^2-y^2 (d) none
   ✓ **Ans. (c) $x^2 - y^2$**

4. Common factors × non- common factors is the formula of..........
   (a) L.C.M (b) H.C.F (c) Remainder (d) none
   ✓ **Ans. (a) L.C.M**

5. How many methods are used to find the L.C.M of a polynomial?
   (a) Three (b) One (c) Two (d) Infinite
   ✓ **Ans. (d) Infinite**

6. The L.C.M of x^2+4x+4 and x^2+5x+6 is..........
   (a) $(x+2)(x+3)$ (b) $(x+2)^2(x+3)$ (c) $(x-2)^2(x+3)$ (d) none
   ✓ **Ans. (b) $(x+2)^2(x+3)$**

7. Relationship between H.C.F and L.C.M is..........
   (a) $A\times B=\frac{H}{L}$ (b) $A\times B=\frac{L}{H}$ (c) $A\times B=H\times L$ (d) none
   ✓ **Ans. (c) $A\times B=H\times L$**

8. The quotient of the two algebraic expressions is called..........
   (a) Algebraic fraction (b) Algebraic expression
   (c) Polynomial (d) none
   ✓ **Ans. (a) Algebraic fraction**

9. The value of $\frac{x+y}{3x+2y}+\frac{x-y}{3x+2y}=$..........
   (a) $\frac{4x}{3x+2y}$ (b) $\frac{2x}{3x+2y}$ (c) $\frac{5x}{3x+2y}$ (d) none
   ✓ **Ans. (b) $\frac{2x}{3x+2y}$**

10. How many methods are used to find square root of algebraic expression?
    (a) One (b) Two (c) Three (d) Infinite
    ✓ **Ans. (b) Two**

UNIT 8:
LINEAR GRAPHS & THEIR APPLICATIONS

Cartesian Plane & Linear Graphs:

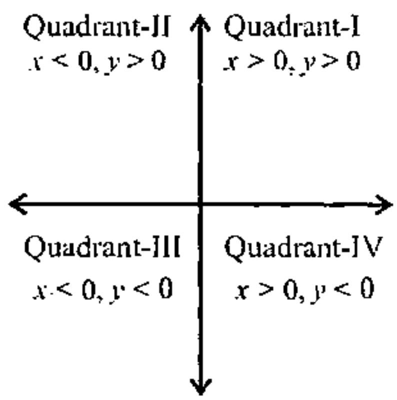
Let $A=\{1,2\}$ and $B=\{3,4\}$ are any two sets. Then the Cartesian product of A and B denoted by $A\times B$ i.e.

$$A\times B=\{(1,3),(1,4),(2,3),(2,4)\}$$

Cartesian product of a real number set is

$$R\times R=\{(a,b)|a,b\in R\}$$

A rectangular coordinate system is formed by two perpendicular number lines, one is horizontal and one is vertical which intersect at the zero point is called *Cartesian plane*. The point of intersection 0(0,0) is called the origin.



It is divided into 4 quadrants.
In first quadrant, $x>0, y>0$
In 2nd quadrant, $x<0, y>0$
In 3rd quadrant, $x<0, y<0$
In 4th quadrant, $x>0, y<0$

Each point which we draw in the xy-plane must be in ordered pair form (a,b). Where a is called abscissa and b is called ordinate. Both (a,b) are called coordinate.

EXAMPLE

Determine the x-coordinate and y-coordinate of the following points. Mention the quadrant in which each points lies. Also plot the points.

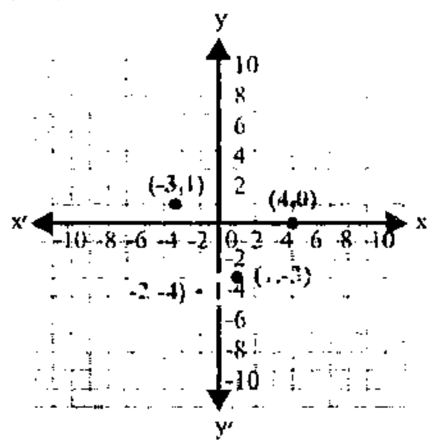
i) (3,1) ii) (-2, -4)
iii) (4,0) iv) (1, -3)

Solution:

i) In the point (-3,1), the x-coordinate is 3 and y-coordinate is 1. The point (-3,1) lies in quadrant II.

ii) In the point (-2,-4), the x-coordinate is -2 and y-coordinate is -4. The point (-2,-4), lies in quadrant III.

iii) In the point (4,0), the x-coordinate is 4 and y-coordinate is 0. The point (4,0) lies on x-axis.



iv) In the point (1,-3), the x-coordinate is 1 and y-coordinate is -3. The point (1,-3), lies in the quadrant IV.

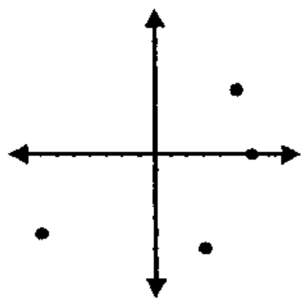
Scale: 1 small square = 1 unit along x-axis
Scale: 1 small square = 1 unit along y-axis

EXAMPLE 5

The points A, B, C and D are shown in the graph. Write the coordinates of the points.

Solution:

The points are A(5,4), B(-6,-5), C(3,-6), D(6,0)

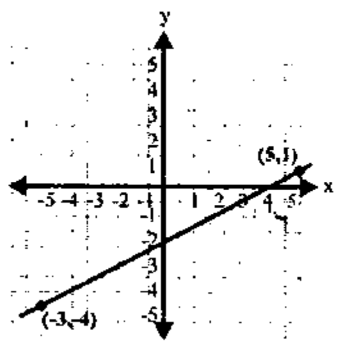


To draw line segment, triangle, rectangle, square and parallelogram by joining the set of given points:

EXAMPLE

Draw a triangle ABC by joining the points A(-2,3), B(4,2), C(1,-3).

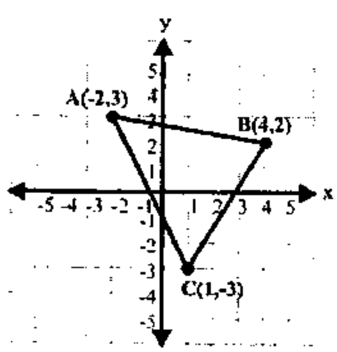
Solution:



Steps of Construction:

a) Plot the points A(-2,3), B(4,2) and C(1,-3) on the xy-plane.

b) Draw the straight line through the points A and B, B and C, A and C to get the sides AB, BC and AC of the triangle ABC. Thus ABC is the triangle.



EXAMPLE

Draw a parallelogram by joining the points O(0,0), A(1,4), B(4,2) and C(3,-2).

Solution:

a) Plot the points O(0,0), A(1,4), B(4,2) and C(3,-2) on the xy-plane.

b) Draw the straight line through O and A, A and B, B and C and O and C to get the sides OA, AB, BC, and OC respectively of the parallelogram or joined all the points by drawing straight line. OABC is a parallelogram.

EXERCISE 8.1

Q1: Determine the x and y coordinates of the following points:

i) $A(-7, 5)$ ii) $B(0, 7)$
iii) $C(-3, 8)$ iv) $D(-3, -3)$
v) $E(10, 12)$

Solution:

i) Given $A(-7,5)$ here
$x=-7,\ y=5$

ii) Given $B(0,7)$ here
$x=0,\ y=7$

iii) Given $C(-3,8)$ here
$x=-3,\ y=8$

iv) Given $D(-3,-3)$ here
$x=-3,\ y=-3$

v) Given $E(10,12)$ here
$x=10,\ y=12$

Q2: Mention the quadrant in which each of the following point lies.

i) $A(-1,\sqrt{2})$ ii) $B(-3, -2)$
iii) $C(5,5)$ iv) $D(3,-5)$
v) $E(-\sqrt{5},\sqrt{7})$

Solution:

i) Given point $A(-1,\sqrt{2})$, it lies in 2nd quadrant.

ii) Given point $B(-3,-2)$, it lies in 3rd

quadrant

iii) Given point $C(5,5)$, it lies in 1st quadrant

iv) Given point $D(3,-5)$, it lies in 4th quadrant

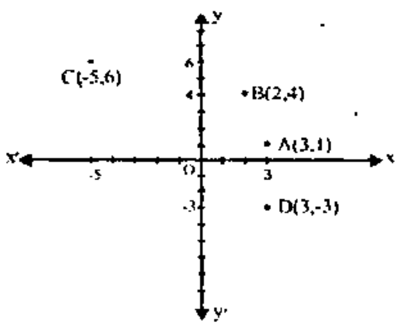
v) Given point $E(-\sqrt{5},\sqrt{7})$, it lies in 2nd quadrant

Q3: Plot the points A, B, C and D on the xy-plane.

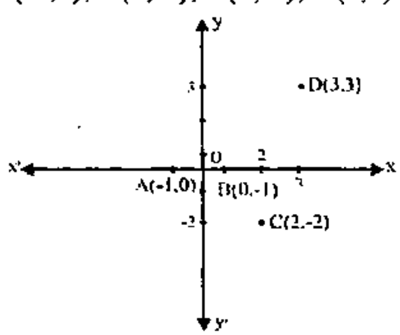
i) $A(3,1), B(2,4), C(-5,6), D(3,-3)$
ii) $A(-1,0), B(0,-1), C(2,-2), D(3,3)$
iii) $A(4,4), B(0,0), C(8,-6), D(-7,5)$

Solution:

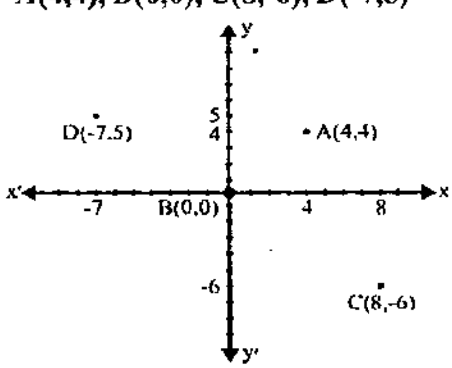
i) $A(3,1), B(2,4), C(-5,6), D(3,-3)$



ii) $A(-1,0), B(0,-1), C(2,-2), D(3,3)$



iii) $A(4,4), B(0,0), C(8,-6), D(-7,5)$

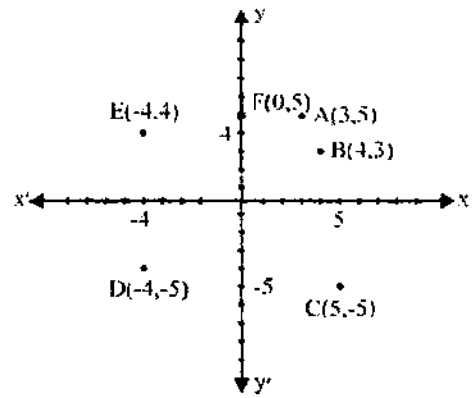


Q4: Plot the points associated with the ordered pairs $A(3,5)$, $B(4,3)$, $C(5,-5)$, $D(-4,-5)$, $E(-4,4)$ and $F(0,5)$.

Solution:

Given points are A(3,5), B(4,3), C(5,-5), D(-4,-5), E(-4,4) and F(0,5)

Their plotting is shown

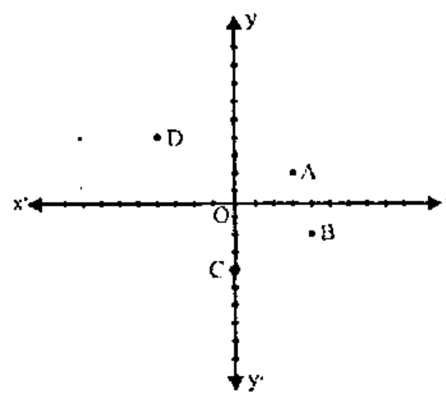


Q5: Write the coordinates of the points A, B, C and D in the given graph.

Solution:

From the graph, the coordinates of points A, B, C and D are written below:

$$A(3,2), B(4,-2), C(0,-3), D(-4,4)$$

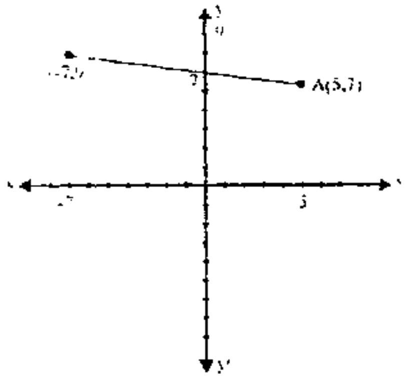


Q6: Draw a line segment by joining the points (5,7) and (-7, 9).

Solution:

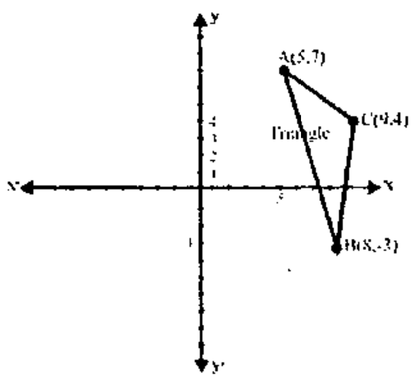
Given points are

A(5, 7) and B(-7, 9) as shown in the figure.



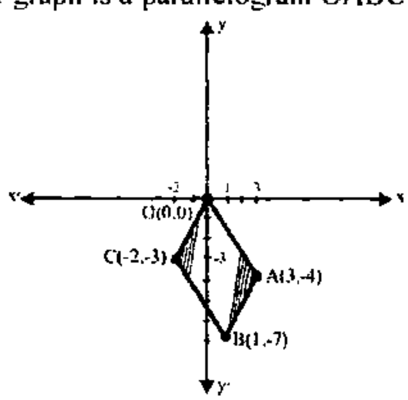
Q7: Draw a triangle *ABC* by joining the points A(5,7), B(8,-3), C(9,4).

Solution:
Given points are
A(5, 7), B(8, -3) and C(9, 4)



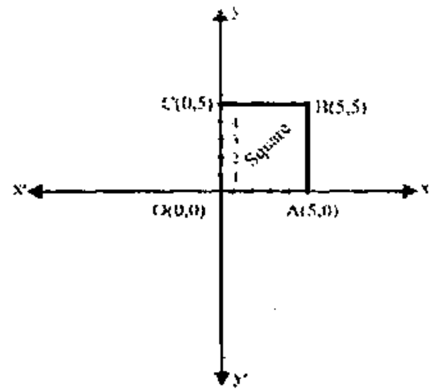
Q8: Draw a parallelogram *OABC* by joining the point O(0,0), A(3,-4), B(1,-7) and C(-2,-3).

Solution:
Given points are O(0,0), A(3,-4), B(1,-7) and C(-2,-3)
Their graph is a parallelogram OABC



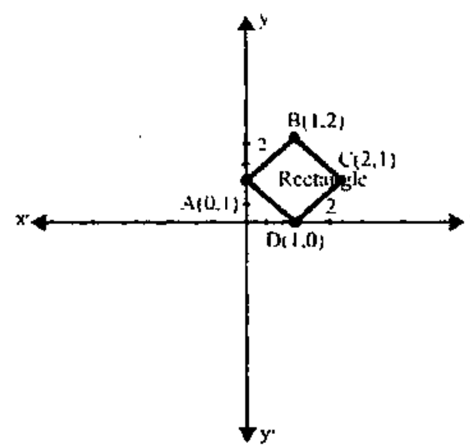
Q9: Join the points O(0,0), A(5,0), B(5,5), C(0,5) to draw a square.

Solution:
Given points are O(0,0), A(5,0), B(5,5), C(0,5). There graph is a square OABC.



Q10: By drawing the graph, that the points A(0,1), B(1,2), C(2,1) and D(1,0) are the vertices of a rectangle.

Solution:
Given points are A(0,1), B(1,2), C(2,1) and D(1,0). Their graph is a rectangle ABCD.



A linear Equation in two Variables:

Definition: An equation of the form $ax + by = c$ (where a, b and c are constants) is called a *linear equation in two variables*. The solution of a linear equation in two variables is a set of ordered pairs satisfying the given linear equation.

EXAMPLE 10

Graph: $3x-5y=10$

Solution:

We first solve for y:

$$3x-5y=-10$$

$$\Rightarrow -y=-3x-10$$

$$\Rightarrow y=\frac{3}{5}x+2$$

Put $x=-5$, we get,

$$y=\frac{3}{5}x+2 \Rightarrow \frac{3}{5}(-5)+2$$

$$=-3+2=-1$$

Similarly,

Put $x=0$, we get $y=2$

Put $x=5$, we get $y=5$

So the ordered pairs (-5, -1), (0,2), (5,5) satisfy the given linear equation.

Note that there are infinite number of ordered pairs that are solutions to the linear equation e.g. (10,8), (-10, -4) etc.

EXAMPLE 11

Draw the graph of $y=-2$.

Solution:

To draw the graph of $y=-2$, we first construct a table showing a few values of x and y.

x	-4	-2	0	2	3
y	-2	-2	-2	-2	-2

Plot the points (-4,2), (-2,-2), (0,-2), (2,-2), (3,-2) in the xy-plane. By joining the points we get the graph of $y=-2$.

In the graph of $y=-2$ the distance of the line is 2 units below the x-axis.

Drawing the graph of the equation of the for $x=a$.

The graph of the equation $x=a$ is a vertical line which is parallel to y-axis.

EXAMPLE 12

Graph the equation $x=4$.

Solution:

We construct a table for some values of x and y.

x	4	4	4	4	4
y	-3	0	1	3	5

We plot the points (4,-3), (4,0) and (4,3) on the xy-plane. The line passing through the points (4,-3), (4,0) and (4,-3) is the graph of $x=4$.

The distance of line from y-axis is 4 units.

Drawing the graph of the equation of the form $y=mx$.

EXAMPLE 13

Graph the equation $y=x$.

Solution:

Here $m=1$, let $x=-3$ then $y=-3$

$x=0$ then $y=0$

$x=1$ then $y=1$

$x=2$ then $y=2$

$x=4$ then $y=4$

x	-3	0	1	2	4
y	-3	0	1	2	4

Now we plot the points (-3,-3), (1,1), (4,4) as shown in the graph. Thus the line joining the given points is the graph of $y=x$.

EXERCISE 8.2

Q1: Determine whether or not each of the following ordered pairs are solutions to the linear equations given:

a) (6,1), $x-5y=1$

b) (5, -10), $2x+y=6$

c) (0, 4), $x-y=2$

d) (-3, 4), $x+3y=2$

Solution:

a) (6,1), $x-5y=1$

Given $x-5y=1 \longrightarrow (1)$

Put $x=6$, $y=1$ in equation (1)

$$6-5(1)=1$$

$$\Rightarrow 6-5=1 \Rightarrow 1=1$$

$L.H.S=R.H.S$ (True)

$\therefore$ (6,1) is the solution of given equation (1)

b) (5, -10), $2x+y=6$

Given $2x+y=6 \longrightarrow (1)$

Put $x=5,\ y=-10$ in equation (1)

$2(5)+(-10)=6$

$\Rightarrow\ 10-10=6 \Rightarrow 0\neq 6$ (False)

$L.H.S \neq R.H.S$

Which is not possible.

$\therefore$ (5,-10) is not the given solution of equation (1)

c) (0, 4), $x-y=2$

Given $x-y=2 \longrightarrow (1)$

Put $x=0,\ y=4$ in equation (1)

$0-4=2$

$\Rightarrow\ -4\neq 2$ (False)

$L.H.S \neq R.H.S$

$\therefore$ (0,4) is not the solution of given equation (1)

d) (-3, 4), $x+3y=2$

Given $x+3y=2 \longrightarrow (1)$

Put $x=-3,\ y=4$ in equation (1)

$-3+3(4)=2$

$\Rightarrow\ -3+12=2$

$\Rightarrow\ 9\neq 2$ (False)

$L.H.S \neq R.H.S$

$\therefore$ (-3,4) is not the solution of equation (1)

Q2: Which of the following points are on the line $3x+2y-6=0$.

(1,1),(4,-3),(3,0),(2,0),(0,2),(0,3),(-2,6)

Solution:

Given equation of line

$3x+2y-6=0 \rightarrow (1)$

Put (1,1) in equation (1)

$3(1)+2(1)-6=0$

$\Rightarrow\ 5-6=0$

$\Rightarrow\ -1\neq 0$ (False)

$L.H.S \neq R.H.S$

$\therefore$ (1,1) is not on the line.

Put $(4,-3)$ in equation (1)

$3(4)+2(-3)-6=0$

$\Rightarrow\ 12-6-6=0$

$\Rightarrow 12-12=0$

$\Rightarrow\ 0=0$ (True)

$L.H.S = R.H.S$

$\therefore$ $(4,-3)$ is on the line.

Put $(3,0)$ in equation (1)

$3(3)+2(0)-6=0$

$\Rightarrow\ 9+0-6=0$

$\Rightarrow\ 3\neq 0$ (False)

$L.H.S \neq R.H.S$

$\therefore$ $(3,0)$ is not on the line.

Put (2,0) in equation (1)

$3(2)+2(0)-6=0$

$\Rightarrow\ 6-6=0 \Rightarrow 0=0$ (True)

$L.H.S = R.H.S$

$\therefore$ $(2,0)$ is on the line.

Check for (0,2), (0,3), (-2,6)

Put (0,2) in equation (1)

$\Rightarrow\ 3(0)+2(2)-6=0$

$\Rightarrow\ 0+4-6=0$

$\Rightarrow\ -2\neq 0$

$\therefore$ Point (0,2) is not on the line.

Put (0,3) in equation (1)

$3(0)+2(3)-6=0$

$0+6-6=0$

$6-6=0$

$0=0$ (True)

L.H.S = R.H.S

$\therefore$ (0,3) is on the line.

Put $(-2,6)$, put $x=-2, y=6$ in eq. (1)

$3(-2)+2(6)-6=0$

$\Rightarrow\ -6+12-6=0$

$\Rightarrow\ 0=0$

$-12+12=0$

As $L.H.S = R.H.S$

The point (-2,6) is on the line.

Q3: Construct a table for four pair of values satisfying the equation $x - y = 4$.

Solution:

Given equation is $x-4=4$

$\Rightarrow x-4=y \Rightarrow y=x-4 \rightarrow (1)$

Put $x=-1$ in equation (1)

$y = -1 - 4 = -5$

Put $x = 0$ in equation (1)

$y = 0 - 4 = -4$

Put $x = 1$ in equation (1)

$y = 1 - 4 = -3$

Put $x = 2$ in equation (1)

$y = 2 - 4 = -2$

$\therefore$ The four pair of values satisfying the given equation are (-1,-5), (0,-4), (1,-3) and (2,-2).

Thus the table is

x	-1	0	1	2
$y = x - 4$	-5	-4	-3	-2

Q4: Draw the graphs of the equations:

a) $y - 2x = 6$ **b)** $y = 1 - x$

c) $y = 2$ **d)** $y = x$

Solution:

a) $y - 2x = 6$

Given $y - 2x = 6$

Or $y = 6 + 2x \longrightarrow (1)$

Put $x = -4$ in equation (1)

$y = 6 + 2(-4)$

$y = 6 - 8 = -2$

Put $x = -3$ in equation (1)

$y = 6 + 2(-3)$

$y = 6 - 6 = 0$

Put $x = -2$ in equation (1)

$y = 6 + 2(-2)$

$y = 6 - 4 = 2$

Put $x = -1$ in equation (1)

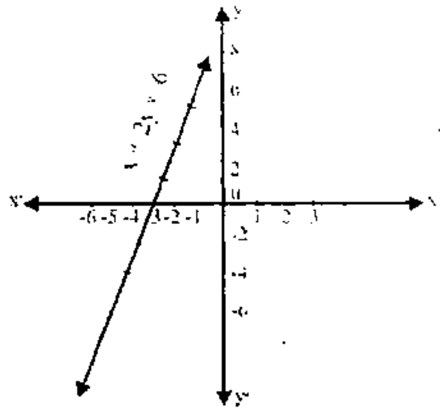
$y = 6 + 2(-1)$

$y = 6 - 2 = 4$

The table is under

x	-4	-3	-2	-1
y	-2	0	2	4

Now the graph of $x - 2y = 6$ is:



b) $y = 1 - x$

Given $y = 1 - x \longrightarrow (1)$

Put $x = -2$ in equation (1)

$y = 1 - (-2)$

$y = 1 + 2 = 3$

Put $x = -1$ in equation (1)

$y = 1 - (-1)$

$y = 1 + 1 = 2$

Put $x = 0$ in equation (1)

$y = 1 - 0 = 1$

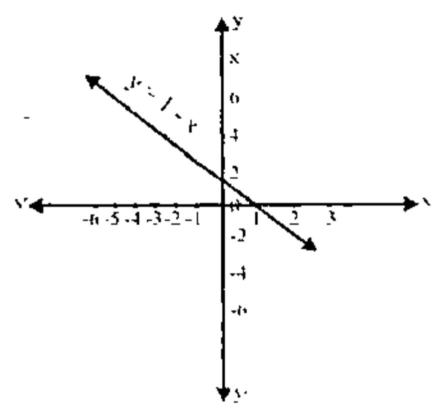
Put $x = 1$

$y = 1 - 1 = 0$

The table is under

x	-2	-1	0	1
y	3	2	1	0

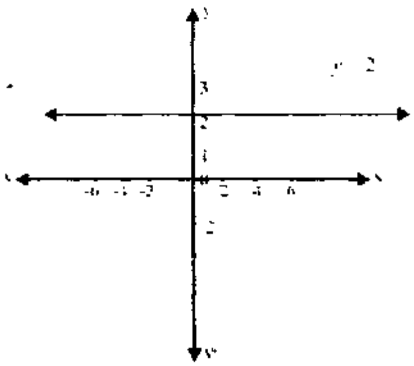
Now the graph of $y = 1 - x$ is:



c) $y = 2$

Given $y = 2$

Since y is constant and the graph of y intercept (y-axis) is 2 and parallel to x-axis.



d) $y = x$

Given $y = x \longrightarrow (1)$

Put $x = -1$ in equation $(1) \Rightarrow y = -1$

Put $x = 0$ in equation $(1) \Rightarrow y = 0$

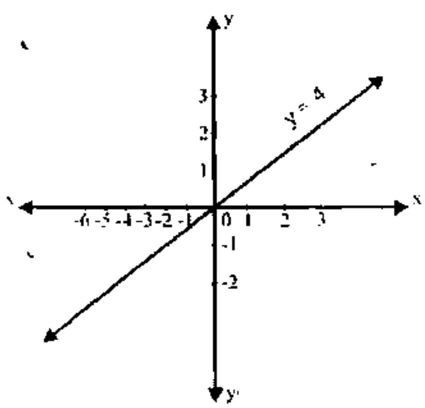
Put $x = 1$ in equation $(1) \Rightarrow y = 1$

Put $x = 2$ in equation $(1) \Rightarrow y = 2$

The table is under

x	0	1	-1	2
y	0	1	-1	2

The graph of $y = x$ is:



Q5: Complete each ordered pair so that it satisfies the given equation.

i) $3x - 7y = 21$; (,15),(14,)(-2,)

ii) $5y + 6x = 30$;(-5,),(,- 6), (,4)

iii) $2y + 9x = 36$; (6,),(0,), (,0)

iv) $4x + 7y = 56$; (,2), (,0), (0,)

Solution:

i) $3x - 7y = 21$; (,15),(14,)(-2,)

Given $3x - 7y = 21 \longrightarrow (1)$

And also given (,15), (14,), (−2,)

Put $y = 15$ in equation (1)

$$3x - 7(15) = 21$$
$$\Rightarrow 3x - 105 = 21$$
$$\Rightarrow 3x = 21 + 105$$
$$\Rightarrow 3x = 126$$
$$\Rightarrow x = \frac{\cancel{126}^{42}}{\cancel{3}} = 42$$

Put $x = 14$ in equation (1)

$$3(14) - 7y = 21$$
$$\Rightarrow 42 - 7y = 21$$
$$\Rightarrow 42 - 21 = 7y \text{ or } 7y = 21$$
$$\Rightarrow y = \frac{\cancel{21}^{3}}{\cancel{7}} = 3$$

Put $x = -2$ in equation (1)

$$3(-2) - 7y = 21$$
$$\Rightarrow -6 - 7y = 21$$
$$\Rightarrow -6 - 21 = 7y \text{ or } 7y = -27$$
$$\Rightarrow y = -\frac{27}{7}$$

Hence the complete ordered pairs are:

$$(42,15),\ (14,3),\ \left(-2, -\frac{27}{7}\right) \quad \text{Ans.}$$

ii) $5y + 6x = 30$;(-5,),(,- 6), (,4)

Given $5y + 6x = 30 \longrightarrow (1)$

And also given (−5,), (,−6), (,4)

Put $x = -5$ in equation (1)

$$5y + 6(-5) = 30$$
$$\Rightarrow 5y - 30 = 30$$
$$\Rightarrow 5y = 30 + 30$$
$$\Rightarrow 5y = 60$$
$$\Rightarrow y = \frac{\cancel{60}^{12}}{\cancel{5}} = 12$$

Put $y = -6$ in equation (1)

$$5(-6) + 6x = 30$$
$$\Rightarrow -30 + 6x = 30$$
$$\Rightarrow 6x = 30 + 30$$
$$\Rightarrow 6x = 60$$
$$\Rightarrow x = \frac{\cancel{60}^{10}}{\cancel{6}} = 10$$

Put $y=4$ in equation (1)

$5(4)+6x=30$

$\Rightarrow 20+6x=30$

$\Rightarrow 6x=30-20$

$\Rightarrow 6x=10$

$\Rightarrow x=\dfrac{\cancel{10}^5}{\cancel{6}_3}=\dfrac{5}{3}$

Hence the complete ordered pairs are:

$(-5,12),(10,-6),\left(\dfrac{5}{3},4\right)$ Ans.

iii) $2y+9x=36$; $(6,\ )$, $(0,\ )$, $(\ ,0)$

Given $2y+9x=36 \longrightarrow (1)$

And also given $(6,\ )$, $(0,\ )$, $(\ ,0)$

Put $x=6$ in equation (1)

$2y+9(6)=36$

$\Rightarrow 2y+54=36$

$\Rightarrow 2y=36-54$

$\Rightarrow 2y=-18$

$\Rightarrow y=-\dfrac{\cancel{18}^9}{\cancel{2}}=-9$

Put $x=0$ in equation (1)

$2y+9(0)=36$

$\Rightarrow 2y+0=36 \Rightarrow 2y=36$

$\Rightarrow y=\dfrac{\cancel{36}^{18}}{\cancel{2}}=18$

Put $y=0$ in equation (1)

$2(0)+9x=36$

$\Rightarrow 0+9x=36 \Rightarrow 9x=36$

$\Rightarrow x=\dfrac{\cancel{36}^4}{\cancel{9}}=4$

Hence the required ordered pairs are:

$(6,-9),(0,18),(4,0)$ Ans.

iv) $4x+7y=56$; $(\ ,2)$, $(\ ,0)$, $(0,\ )$

Given $4x+7y=56 \longrightarrow (1)$

And $(\ ,2)$, $(\ ,0)$, $(0,\ )$

Put $y=2$ in equation (1)

$\Rightarrow 4x+7(2)=56$

$\Rightarrow 4x+14=56$

$\Rightarrow 4x=56-14 \Rightarrow 4x=42$

$\Rightarrow x=\dfrac{\cancel{42}^{21}}{\cancel{2}}=\dfrac{21}{2}$

Put $y=0$ in equation (1)

$\Rightarrow 4x+7(0)=56$

$\Rightarrow 4x+0=56 \Rightarrow 4x=56$

$\Rightarrow x=\dfrac{\cancel{56}^{14}}{\cancel{4}}=14$

Put $x=0$ in equation (1)

$\Rightarrow 4(0)+7y=56$

$\Rightarrow 0+7y=56 \Rightarrow 7y=56$

$\Rightarrow y=\dfrac{\cancel{56}^8}{\cancel{7}}=8$

Hence the complete ordered pairs are:

$\left(\dfrac{21}{2},2\right),(14,0),(0,8)$ Ans.

Q6: The weight in kilogram and age in years of a person is expressed by the equation $y=2x$, where y (in kg) and x (in years). Draw the age-weight graph from the values of the following table:

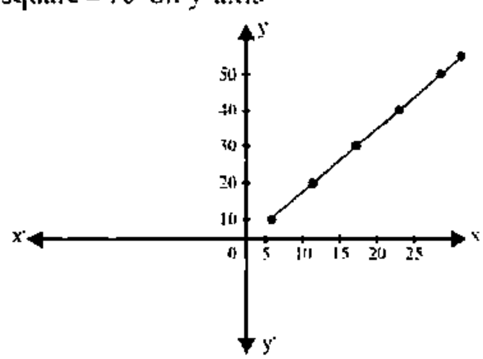
x	5	10	15	20	25	30
y	10	20	30	40	50	60

Scale:

Using the above table and draw its graph in xy-plane as follow:

Let 1 square = 5 on x-axis

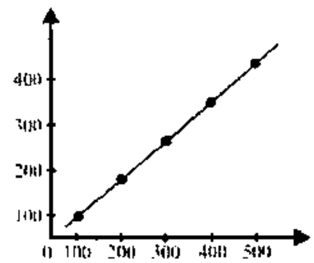
1 square = 10 on y-axis



Q7: The graph shows the relations between the units of electricity consumed and the total cost of the electricity bill (i) Find the cost of the bill if 300 units

(n) are consumed. (ii) The number of units used when the bill is Rs. 1500.

Solution:



Cost of the bill is using the graph we shall answer the following:

i) We can see from the graph that if 300 units are consumed, then

Cost $(c) = Rs.2500$

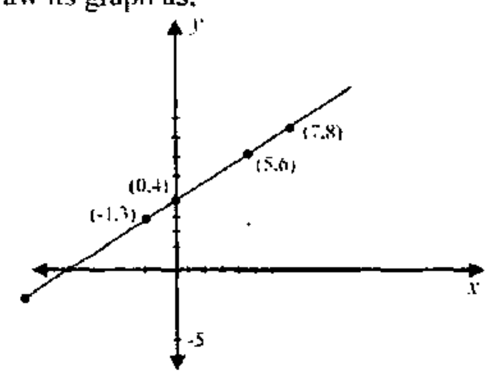
ii) When the bill is Rs. 1500, then the number of units used = 150 units.

Q8: Draw the graph from the following table by using a suitable scale.

x	0	-1	5	7	-4
y	4	3	6	8	-5

Solution:

Using values of the given table we can draw its graph as;

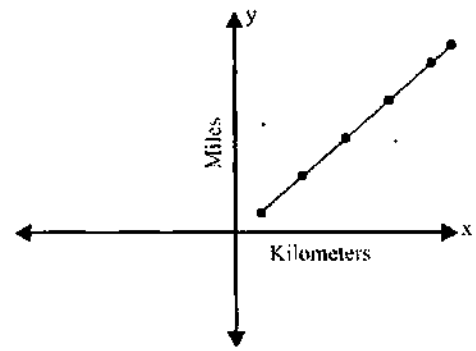


Conversion of Graphs:

If two quantities in a relation either both are increasing or decreasing then the graph of the relation will be straight line.

Conversion Graph of Miles into Kilometers:

Let the variable 'M' denotes the distance in miles indicated along y-axis and 'K' denotes along x-axis the distance in kilometers. The relation is represented by the equation $M = f(K)$.



$\because 1M = 1.60\,km$

M	1	2	3	4	5
Km	1.60	3.20	4.80	6.40	8.00

Scale:

1 mile = 1 unit along y-axis

$1\,Km = 1$ unit along x-axis

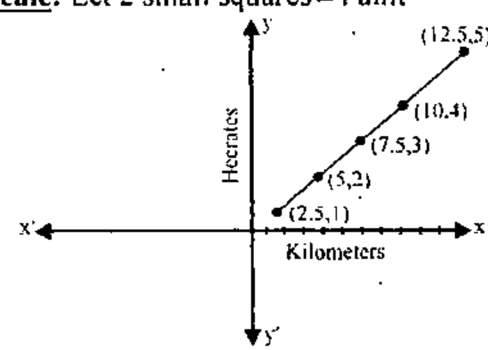
Conversion of Hectares into Acres:

Note: $1H = 2.5A$ where H = hectares, A = acres.

First we write a table in which hectare are converted into acre.

H	1	2	3	4	5	6
A	2.5	5	7.5	10	12.5	15

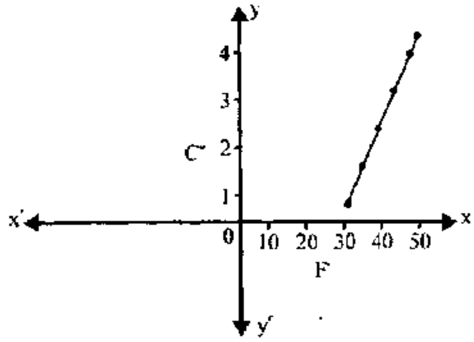
Scale: Let 2 small squares = 1 unit



Conversion of Degree Celsius into Degree Fahrenheit:

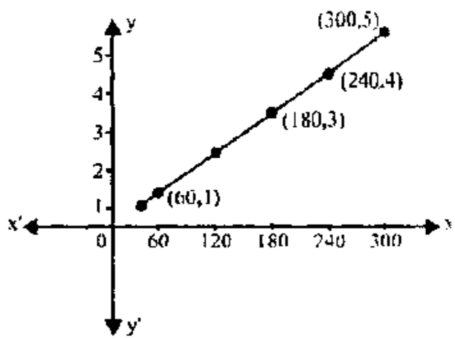
Note: $F^o = \frac{9}{5}C^o + 32$

C^o	1	2	3	4	5
F^o	33.8 = 34	35.6 = 36	37.4 = 37	39.2 = 39	41



Conversion of Pakistani Currency into Foreign Currency:

US #	1	2	3	4
PKR	160	320	480	640



EXERCISE 8.3

Q1: Using the conversion formula 1 mile $=1.6\,km$. Draw the conversion graph of Miles-Kilometers if distance in miles are given as:
1M, 3M, 4M, 5M (M is used for miles)

Solution:

Since $1M = 1.60\,Km$

The table values are;

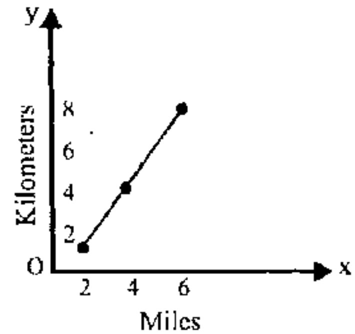
x (M)	1	3	4	5
y (Km)	1.60	4.80	6.40	8.00

Scale:

Take $1M = 1$ unit along x-axis

$1\,Km = 1$ unit along y-axis

The graph is shown below:



Q2: Draw the miles-kilometers graph. If distance in kilometers are given as 1km, 2km, 3km, 4km and 5km.

Solution:

As $1.60\,Km = 1\,M$

$$\Rightarrow 1\,Km = \frac{1}{1.60} = 0.625$$

Then

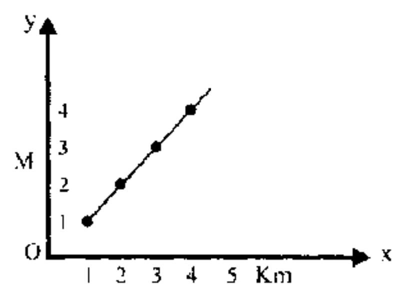
$x(Km)$	1	2	3	4	5
$y(M)$	0.625	1.25	1.875	2.5	3.125

Scale:

Take $1\,Km = 1$ unit along x-axis

$1M = 1$ unit along y-axis

The graph is shown below:



Q3: Given that 1 hectare = 2.5 acres (approximately), draw a conversion graph of hectare. Acre from the given values of hectare $2H$, $4H$, $8H$.

Solution:

Given $1H = 2.5A$, then

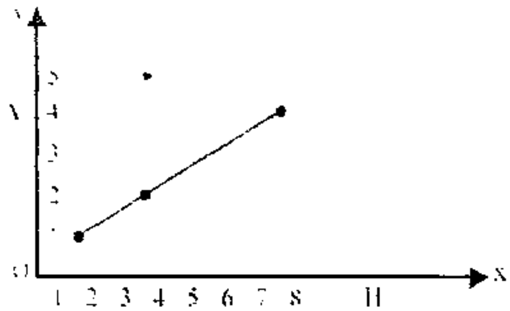
H	2	4	8
A	5	10	20

Scale:

$1H = 1$ unit along x-axis

$4A = 1$ unit along y-axis

The graph is shown below:



Q4: Convert the following temperature given in Celsius degree into Fahrenheit and then draw its graph. Degree Celsius=0°C, 2°C, 3°C. Conversion formula is: $F^o = \frac{9}{5}C^o + 32$

Solution: Degree Celsius $= 0^o, 2^o, 3^o$

Given $F^o = \frac{9}{5}C^o + 32$

Put $C^o = 0$

$\Rightarrow F^o = \frac{9}{5}(0) + 32 = 32^o$

Put $C^o = 2$

$\Rightarrow F^o = \frac{9}{5}(2) + 32 = \frac{18+160}{5}$

$\Rightarrow F^o = \frac{178}{5} = 35.6$

Put $C^o = 3$

$\Rightarrow F^o = \frac{9}{5}(3) + 32 = \frac{27+160}{5}$

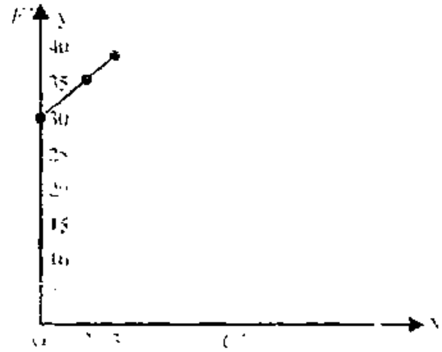
$\Rightarrow F^o = \frac{187}{5} = 37.4$

The table values are:

C^o	0	2	3
F^o	32	$35.6 = 36$	$37.4 = 37$

Now the graph let $1C^o = 1$ unit along x-axis

$5F^o = 1$ unit along y-axis



Q5: Draw the following conversion graph:

(i) PKR (Pakistan rupees) – US$ (1$, 3$, 5$)

(ii) PKR (Pakistani rupees) – GB£ (1£, 2£, 3£)

"$" is used for US dollar and "£" is used for pound sterling, where 1$ = Rs. (60) approximately.

1GB£=Rs. 100 (approx)

Solution:

i) Given 1$ = Rs. 60 then.

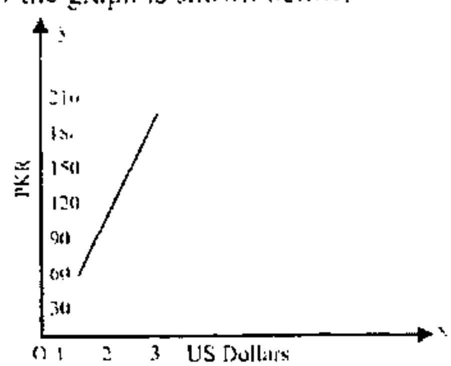
x(US$)	1	2	3
y(PKR)	60	120	180

Scale:

Let 1$ = 1 unit along x-axis

30 PKR = 1 unit along y-axis

Now the graph is shown below:



ii) Given 1GBB=Rs. 100 then,

x(US$)	1	2	3
y(PKR)	100	200	300

Scale:

i) Let 1GBB = 1 unit along x-axis
50PKR = 1 unit along y-axis

Now the graph is shown below:

Q6: Solve the following simultaneous equations by using graphical method.

i) $2x + y = 3,\ x - y = 0$

ii) $y = 2x + 2,\ y = x - 1$

iii) $x + 4y = 5,\ 2x + 3y = 0$

iv) $3x + 5y = 2,\ 3x + 5y = 8$

v) $3x - 2y = 13,\ 2x + 3y = 13$

Solution:

i) $2x + y = 3,\ x - y = 0$

Given $2x + y = 3 \longrightarrow (1)$

$x - y = 0 \longrightarrow (2)$

The tables are constructed showing the values of x and y satisfying both equations.

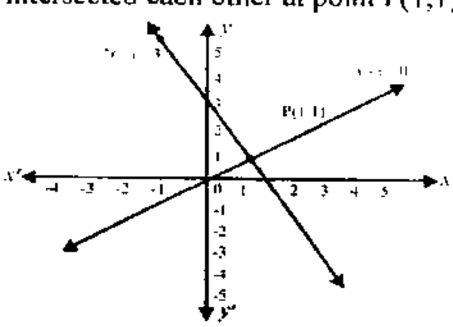
Table for $2x + y = 3$:

x	-1	0	1	2	3	4
y	5	3	1	-1	-3	-5

Table for $x - y = 0$

x	-1	0	1	2	3	4
y	-1	0	1	2	3	4

Take the points from both tables on the graph and then draw straight line joining all the points. In the graph, the two lines are intersected each other at point P(1,1).



Hence the solution of (1) and (2) is the point P(1,1).

ii) $y = 2x + 2,\ y = x - 1$

Given $y = 2x + 2 \longrightarrow (1)$

$y = x - 1 \longrightarrow (2)$

The following tables are constructed showing the values of x and y satisfying both equations.

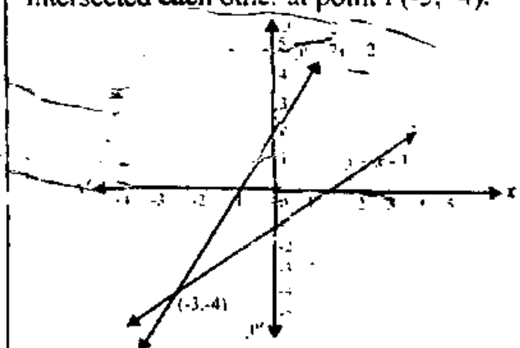
Table for $y = 2x + 2$

x	-3	-2	-1	0	1	2
y	-4	-2	0	2	4	6

Table for $y = x - 1$

x	-3	-2	-1	0	1	2
y	-4	-3	-2	-1	0	1

Take the points from both tables on the graph and then draw straight line joining all the points. In the graph, the two lines are intersected each other at point P(-3, -4).



Hence solution is the point (-3,-4).

iii) $x + 4y = 5,\ 2x + 3y = 0$

Given $x + 4y = 5 \longrightarrow (1)$

$2x + 3y = 0 \longrightarrow (2)$

The following tables are constructed showing the values of x and y satisfying both equations.

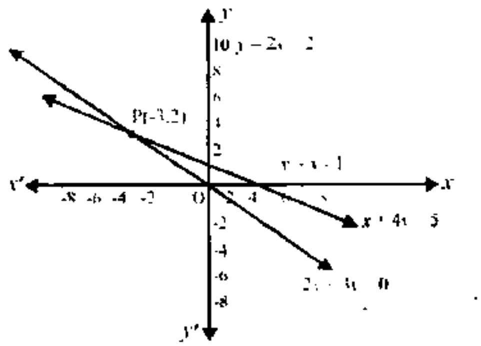
Table for $x + 4y = 5$

x	-7	-3	1	5	9	13
y	3	2	1	0	-1	-2

Table for $2x + 3y = 0$

x	-3	0	3	6	9	12
y	2	0	-2	-4	-6	-8

Take the points from both tables on the graph and then draw straight line joining all the points. In the graph, the two lines are intersected each other at point (-3,2).



Hence the point (-3, 2) is the required solution.

iv) $3x+5y=2,\ 3x+5y=8$

Given $3x+5y=2 \longrightarrow (1)$

$3x+5y=8 \longrightarrow (2)$

The following tables are formed showing the values of x and y satisfying both equations.

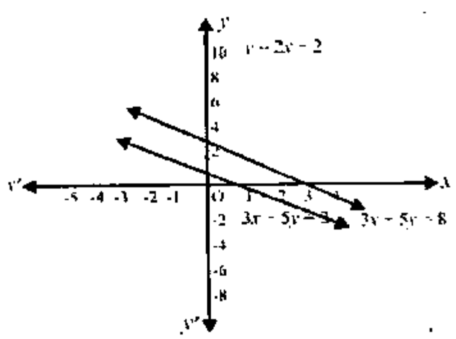
Table for $3x+5y=2$

x	-1	0	1	2	3	4
y	1	$\frac{2}{5}$	$\frac{-1}{5}$	$\frac{-4}{5}$	$-\frac{7}{5}$	-2

Table for $3x+5y=8$

x	-1	0	1	2	3
y	$\frac{11}{5}=2.5$	$\frac{8}{5}=1.6$	1	$\frac{2}{5}=.4$	$-\frac{1}{5}$

Take the points of both tables on the graph and then draw straight line joining all the points. Keep in mind that in the graph the two lines does not intersect each other.



Because they are parallel.

As two lines does not intersect each other, hence the solution set is $=\{\ \}$.

v) $3x-2y=13,\ 2x+3y=13$

Given $3x-2y=13 \longrightarrow (1)$

$2x+3y=13 \longrightarrow (2)$

The following tables are constructed showing the values of x and y satisfying both equations.

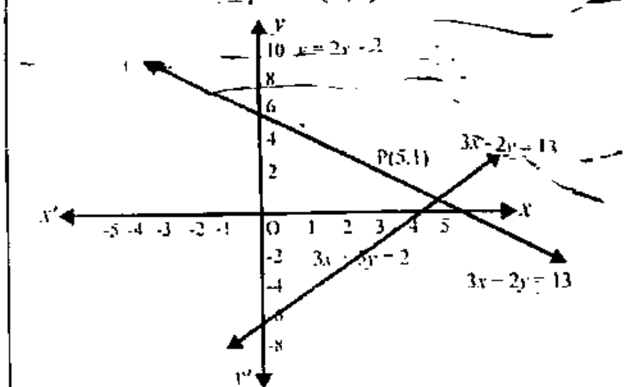
Table for $3x-2y=13$

x	-1	1	3	5	7
y	-8	-5	-2	1	4

Table for $2x+3y=13$

x	-4	-1	2	5	8
y	7	5	3	1	-1

Plot the points from both tables on the graph and then draw straight line joining all the points corresponding to the given equations. In the graph, the two lines intersect each other at point (5,1).



Hence the solution set is $\{(5,1)\}$. Ans.

Review Exercise 8

Q1: Select the correct answer.

i) The point (5, -2) is located in:
(a) Quadrant I
(b) Quadrant II
(c) Quadrant II
✓**(d) Quadrant IV**

ii) The two coordinate axes intersect at an angle of:
(a) 30° (b) 60°
✓**(c) 90°** (d) 45°

iii) The point (-3,8) is located in:
(a) Quadrant I
✓**(b) Quadrant II**
(c) Quadrant III
(d) Quadrant IV

iv) The lines represented by the equations $x+y=1$ and $x+y=4$ are:
✓**(a) Parallel**
(b) Inclined
(c) Intersecting
(d) Perpendicular

v. The point (-6,-6) is located in:
(a) Quadrant I
(b) Quadrant II
✓**(c) Quadrant III**
(d) Quadrant IV

vi. The line $x=a$ where a is a real number is parallel to:
✓**(a) y-axis**
(b) x-axis
(c) Both x-axis and y-axis
(d) Neither x-axis nor y-axis

vii. The point (2,11) is located in:
✓**(a) Quadrant I**
(b) Quadrant II
(c) Quadrant III
(d) Quadrant IV

viii. The solution set of the lines $y=2$ and $y=3$ is:
(a) {5,3} (b) {4,0}
(c) {0,0} ✓**(d) { }**

ix. The line $y=5$ is parallel to:
(a) y-axis
✓**(b) x-axis**
(c) Both x-axis and y-axis
(d) Neither y-axis nor x-axis

x. Solution set of simultaneous equations $y=x+1$, $y=2x-2$
(a) $x=2, y=4$
✓**(b) $x=3, y=4$**
(c) $x=2, y=4$
(d) $x=2, y=4$

Q2: Determine the x-coordinate and y-coordinate of the following points. Also mention the quadrant in which each point lies.

i) (2,3) ii) (-4, 5)
iii) (4,0)

Solution:

i) (2,3)
Given point (2,3)
x-coordinate is 2
y-coordinate is 3
Since in (2,3) the x-coordinate is positive and the y-coordinate is also positive. So the point (2,3) lies in the 1^{st} quadrant.
Hence x-coordinate is 2 and y-coordinate is 3 and the point (2,3) lies in the 1^{st} quadrant.

ii) (-4, 5)
Given point (-4, 5)
x-coordinate is -4
y-coordinate is 5
Since in (-4, 5) the x-coordinate is negative and the y-coordinate is positive. So the point (-4, 5) lies in the 2^{nd} quadrant.
Hence x-coordinate is -4 and y-coordinate is 5 and the point (-4,5) lies in the 2^{nd} qua-

drant.

iii) (4, 0)

Given point is (4,0)

x-coordinate is 4

y-coordinate is 0

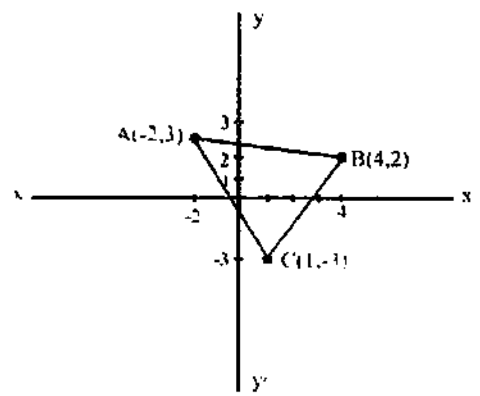
Since in (4,0) the x-coordinate is positive and the y-coordinate is zero. So the point (4,0) lies on x-axis.

Hence x-coordinate is 4 and y-coordinate is 0 and the point (4,0) lies on x-axis.

Q3: Draw a triangle ABC by joining the points A(-2,3), B(4,2), C(1,-3).

Solution:

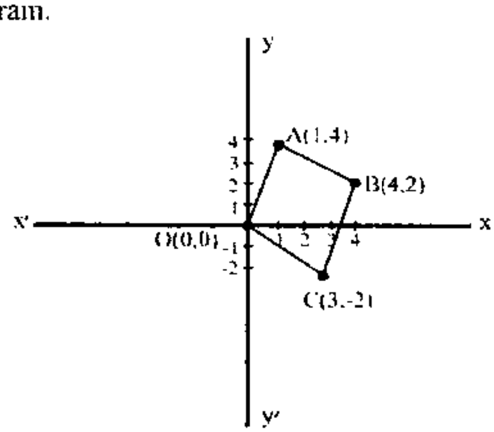
Given points are A(-2,3), B(4,2) and C(1,-3). Their graph is a triangle.



Q4: Draw a parallelogram by joining the points O(0,0), A(1,4), B(4,2) and C(3,-2).

Solution:

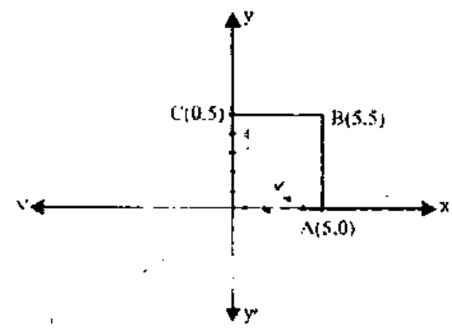
Given points are O(0,0), A(1,4), B(4,2) and C(3,-2). Their graph is a parallelogram.



Q5: By joining the points O(0,0), A(5,0), B(5,5) and C(0,5), draw a square.

Solution:

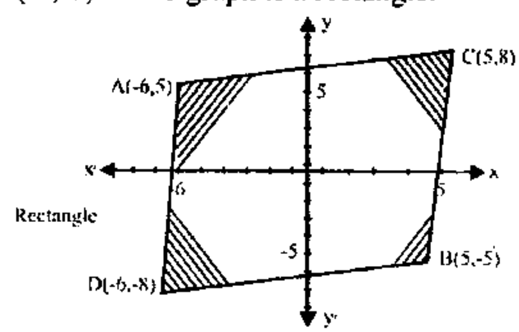
Given points are O(0,0), A(5,0), B(5,5), C(0,5). Their graph is a square.



Q6: Draw a rectangle by joining the points A(-6,5), B(5,-5), C(5,8) & D(-6, -8)

Solution:

Given points are A(-6,5), B(5,-5), C(5,8), D(-6,-8). Their graph is a rectangle.



Q7: Draw the graph of the equation $x + y = 4$.

Solution:

Given that $x + y = 4$

$\Rightarrow y = 4 - x \rightarrow (1)$

Put $x = -1$

$\Rightarrow y = 4 - (-1)$

$= 4 + 1 = 5$

Put $x = 0$

$\Rightarrow y = 4 - 0 = 4$

Put $x = 1$

$\Rightarrow y = 4 - 1 = 3$

Put $x = 2$

$\Rightarrow y = 4 - 2 = 2$

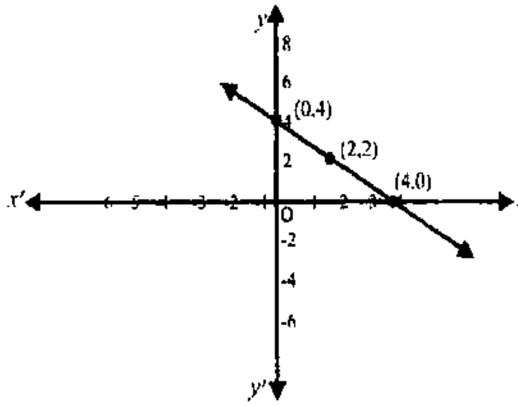
Put $x=3$ then $y=4-3=1$

Put $x=4$ then $y=4-4=0$

The table is under:

x	-1	0	1	2	3	4
y	5	4	3	2	1	0

The graph of $x+y=4$ is:

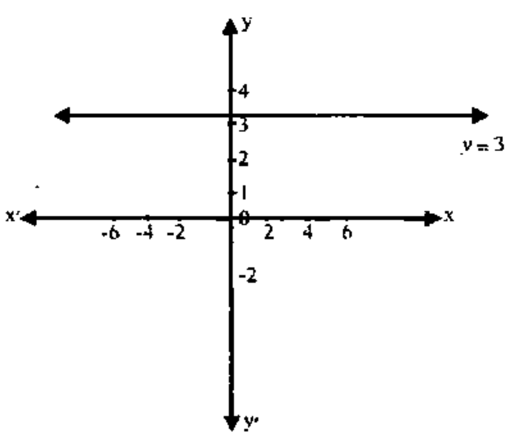


Q8: Draw the graph of $y=3$.

Solution:

Given $y=3$

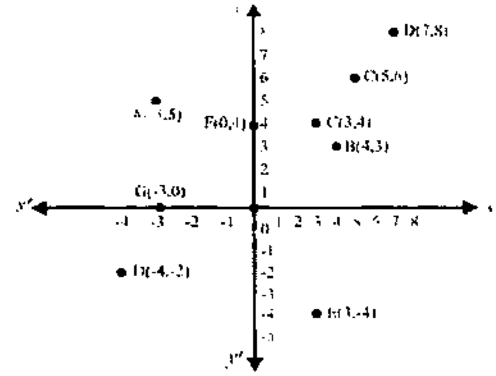
Since y is constant function and its graph will be parallel to x-axis, this type of line is called horizontal line which intersects y-axis at $y=3$.



Q9: Draw a graph from the following table by using a suitable scale.

x	-3	4	3	-4	3	0	-3	0
y	5	3	4	-2	-4	4	0	0

Solution: The graph of the given table can be shown in dots.



Think:

Q10: Solve the following system of equations graphically, $x+y=2$, $x-y=4$

Solution:

Given $x+y=2 \rightarrow (1)$

$x-y=4 \rightarrow (2)$

The tables are constructed showing the values of x and y satisfying both equations.

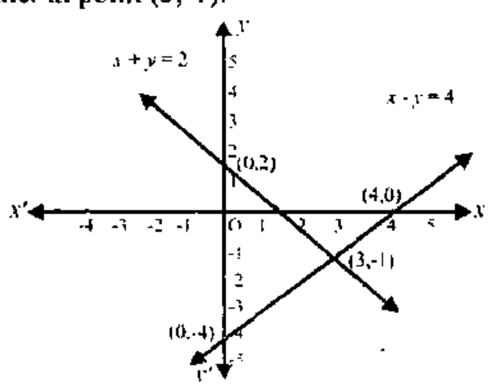
Table for $x+y=2$ or $y=2-x$

x	-1	0	1	2	3	4
y	3	2	1	0	-1	-2

Table for $x-y=4$ or $y=x-4$

x	-1	0	1	2	3	4
y	-5	-4	-3	-2	-1	0

Plot the points from both tables on the graph and then draw straight line joining all the points. The two lines intersect each other at point (3,-1).



Hence the solution of equation (1) and equation (2) is the point (3,-1).

***

Additional MCQs of Unit 8: Linear Graphs and their Applications

1. $A\times B=\{(a,b):a\in A\wedge b\in B\}$ is called..........
   (a) Cartesian product (b) Relation (c) Function (d) Ordered pair
   ✓**Ans. (a) Cartesian product**

2. The ordered pair $(a,b)=$ ....
   (a) (b,a) (b) $(-a,-b)$ (c) $(-b,-a)$ (d) none
   ✓**Ans. (d) none**

3. The two dimensional coordinate system is called..........plane.
   (a) Real (b) Space (c) Cartesian (d) none
   ✓**Ans. (c) Cartesian**

4. If $A=m$ and $b=n$ elements then $A\times B=$ ....elements.
   (a) $m+n$ (b) 2^{mn} (c) $m\times n$ (d) none
   ✓**Ans. (c) $m\times n$**

5. The graph of $ax+by+c=0$ is..........
   (a) Circle (b) Straight line (c) Rectangle (d) none
   ✓**Ans. (b) Straight line**

6. The point $P(2,-3)$ lies in..........quadrant.
   (a) 1^{st} (b) 2^{nd} (c) 3^{rd} (d) 4^{th}
   ✓**Ans. (d) 4^{th}**

7. Which of the following points lies in 3^{rd} quadrant?
   (a) $(2,-3)$ (b) $(-2,-3)$ (c) $(-2,3)$ (d) none
   ✓**Ans. (b) (-2, -3)**

8. $ax+by=c$ if $a\neq 0$ and $b\neq 0$ is called linear equation in..........variables.
   (a) One (b) Three (c) Two (d) none
   ✓**Ans. (c) Two**

9. The graph of $y=a$ is..........line.
   (a) Vertical (b) Horizontal (c) Straight (d) none
   ✓**Ans. (b) Horizontal**

10. $x=a$ represents..........line.
    (a) Vertical (b) Horizontal (c) Parallel (d) none
    ✓**Ans. (a) Vertical**

UNIT 9:
INTRODUCTION TO COORDINATE GEOMETRY

In 1637, the French mathematician Rene Descartes introduced this geometry which is known as analytic geometry.

Definition: That geometry in which we study algebra as well as geometry is called analytic geometry.

Distance Formula between two Points:

Suppose we have two points a and b on the real line;

i) The directed distance from b to a is $b-a$.

ii) The distance between a and b is $a-b$.

iii) The distance between a and b is $|a-b|$ or $|b-a|$. The distance can never be negative.

Distance between two points in a plane

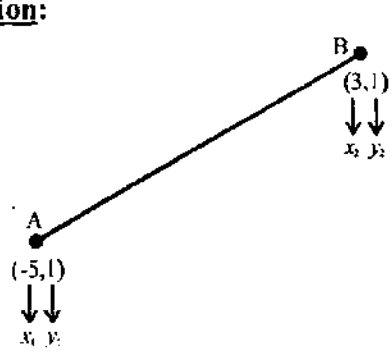
The distance between two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is given by the formula

$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

EXAMPLE 3

Find the distance between points A(-5, 1) and B(3,1).

Solution:



Let $x_1=-5,\ y_1=1$

$x_2=3,\ y_1=1$

Using distance formula, we have

$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$d=\sqrt{\{3(-5)\}^2+(1-1)^2}$$

$$=\sqrt{(3+5)^2+(1-1)^2}$$

$$=\sqrt{(8)^2+(0)^2}$$

$$=\sqrt{64+0}$$

$$=\sqrt{64}=8 \quad \text{Ans.}$$

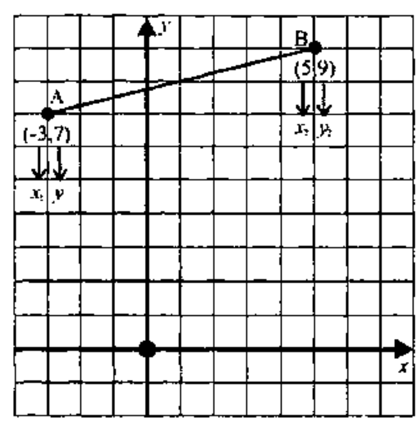
EXAMPLE 4

Plot the points (-3,7) and (5,9) and find the distance between them.

Solution:

Let $A=(-3,7)$ and $B=(5,9)$ and suppose d is the distance between them, using the formula, then

$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$



We have,

$$d=\sqrt{(-5+3)^2+(9-7)^2}$$

$$=\sqrt{(-8)^2+(2)^2}$$

$$=\sqrt{64+4}=\sqrt{68}$$

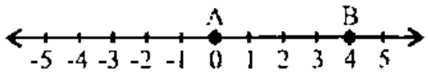
$$=\sqrt{2\times2\times17}$$

$$=2\sqrt{17} \quad \text{Ans.}$$

EXERCISE 9.1

Q1: Find the length of AB in the following figures:

i)



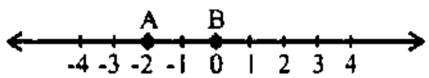
Solution:

Length of $AB = |b-a|$

Where $a=0$ and $b=4$

$= |4-0| = 4$

ii)



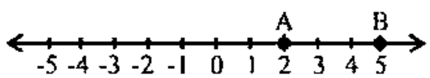
Solution:

Length of $AB = |b-a|$

Where $b=0$ and $a=-2$

$= |0-(-2)| = |2| = 2$

iii)



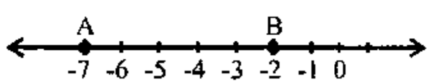
Solution:

Length of $AB = |b-a|$

Where $a=2$ and $b=5$

$= |5-2| = |3| = 3$

iv)



Solution:

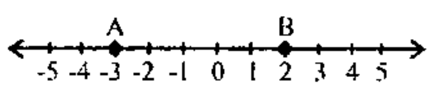
Length of $AB = |b-a|$

Where $a=-7$ and $b=-2$

$= |b-a| = |(-2)-(-7)|$

$= |-2+7| = |5| = 5$

v)



Solution:

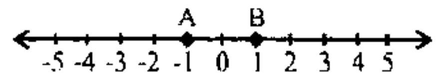
Length of $AB = |b-a|$

Where $a=-3$ and $b=2$

$= |4-(-3)|$

$= |4+3| = |7| = 7$

vi)



Solution:

Length of $AB = |b-a|$

Where $a=-1$ and $b=1$

$= |1-(-1)| = |1+1|$

$= |2| = 2$

Q2: Find the distance between each pair of points:

i) (1,1), (3,3) ii) (1,2), (4,5)
iii) (2,-2), (2,-3) iv) (3,-5), (5, -7)

Solution:

i) (1,1), (3,3)

Let $(x_1, y_1) = (1,1)$ and $(x_2, y_2) = (3,3)$

Then from distance formula

$$d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$

Putting the values

$$= \sqrt{(3-1)^2 + (3-1)^2}$$

$$= \sqrt{(2)^2 + (2)^2}$$

$$= \sqrt{4+4} = \sqrt{8}$$

$$= \sqrt{2\times 2\times 2} = 2\sqrt{2} \quad \text{Ans.}$$

ii) (1,2), (4,5)

Let $(x_1, y_1) = (1,2)$ and $(x_2, y_2) = (4,5)$

Then using distance formula

$$d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$

Putting the values

$$d = \sqrt{(4-1)^2 + (5-2)^2}$$

$$= \sqrt{(3)^2 + (3)^2}$$

$$= \sqrt{9+9} = \sqrt{18}$$

$$= \sqrt{9\times 2} = 3\sqrt{2} \quad \text{Ans.}$$

iii) (2,-2), (2,-3)

Let $(x_1, y_1) = (2,-2)$ and $(x_2, y_2) = (2,-3)$

Then use distance formula

$$d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$

Putting the values

$=\sqrt{(2-2)^2+(-3+2)^2}$

$=\sqrt{0+(-1)^2}=\sqrt{1}=1$ Ans.

iv) (3,-5), (5,-7)

Let $(x_1,y_1)=(3,-5)$ and $(x_2,y_2)=(5,-7)$

Then use distance formula

$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

Putting the values

$=\sqrt{(5-3)^2+(-7+5)^2}$

$=\sqrt{(2)^2+(-2)^2}$

$=\sqrt{4+4}=\sqrt{8}$

$=\sqrt{2\times2\times2}=2\sqrt{2}$ Ans.

Q3: Given points O(0,0), A(3,4), B(-5,12), C(15,-8), D(11,-3), E(-9,-4). Determine the lengths of the following segments.

i) $\overline{OA}$ **ii)** $\overline{OB}$ **iii)** $\overline{OC}$

iv) $\overline{AD}$ **v)** $\overline{AB}$ **vi)** $\overline{AC}$

vii) $\overline{BE}$

Solution:

i) $\overline{OA}$

Given O(0,0), A(3,4)

The distance formula

$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}\rightarrow(1)$

$\overline{OA}=A-O$

Put $(x_1,y_1)=(0,0)$ and $(x_2,y_2)=(3,4)$ in equation (1)

$=\sqrt{(3-0)^2+(4-0)^2}$

$=\sqrt{(3)^2+(4)^2}$

$=\sqrt{9+16}=\sqrt{25}=5$ Ans.

ii) $\overline{OB}$

$\overline{OB}=B-O$

$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}\rightarrow(1)$

Put $(x_1,y_1)=(0,0)$ and $(x_2,y_2)=(-5,12)$ in equation (1)

$=\sqrt{(-5-0)^2+(12-0)^2}$

$=\sqrt{(-5)^2+(12)^2}$

$=\sqrt{25+144}$

$=\sqrt{169}=13$ Ans.

iii) $\overline{OC}$

$\overline{OC}=C-O$

The distance formula is

$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}\rightarrow(1)$

$=\sqrt{(15-0)^2+(-8-0)^2}$

$=\sqrt{(15)^2+(-8)^2}$

$=\sqrt{225+64}=\sqrt{289}$

$=\sqrt{17\times17}=17$ Ans.

iv) $\overline{AD}$

$\overline{AD}=D-A$

$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}\rightarrow(1)$

Put $(x_1,y_1)=(3,4)$ and $(x_2,y_2)=(11,-3)$ in equation (1)

$=\sqrt{(11-3)^2+(-3-4)^2}$

$=\sqrt{(8)^2+(-7)^2}$

$=\sqrt{64+49}=\sqrt{113}$ Ans.

v) $\overline{AB}$

$\overline{AB}=B-A$

$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}\rightarrow(1)$

Put $(x_1,y_1)=(3,4)$ and $(x_2,y_2)=(-5,12)$ in equation (1)

$=\sqrt{(-5-3)^2+(12-4)^2}$

$=\sqrt{(-8)^2+(8)^2}$

$=\sqrt{64+64}=\sqrt{128}$

$=\sqrt{2\times2\times2\times4\times4}$

$=2\times4\sqrt{2}=8\sqrt{2}$ Ans.

vi) $\overline{AC}$

$\overline{AC}=C-A$

$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}\rightarrow(1)$

Put $(x_1,y_1)=(3,4)$ and $(x_2,y_2)=(15,-8)$ in equation (1)

$=\sqrt{(15-3)^2+(-8-4)^2}$

$=\sqrt{(12)^2+(-12)^2}$

$=\sqrt{144+144}=\sqrt{288}$

$=\sqrt{12\times12\times2}=12\sqrt{2}$ Ans.

vii) $\overline{BE}$

$\overline{BE}=E-B$

$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}\rightarrow(1)$

Put $(x_1,y_1)=(-5,12)$ & $(x_2,y_2)=(-9,-4)$ in equation (1)

$=\sqrt{(-9+5)^2+(-4-12)^2}$

$=\sqrt{(-4)^2+(-16)^2}$

$=\sqrt{16+256}=\sqrt{272}$

$=\sqrt{2\times2\times2\times2\times17}$

$=4\sqrt{17}$ Ans.

Collinear and Non-Collinear Points:

Three or more points P_1, P_2, P_3,.... are said to be collinear if they lie on the same line. If the points are not collinear then they are called non-collinear.

Collinear Points *Non-Collinear Points*

EXAMPLE 6

Prove that the points A(5, -2), B(1,2) and C(-2, 5) are collinear.

Solution:

Given $A=(5,-2), B=(1,2)$ & $C=(-2,5)$

Using distance formula, we can write

$|AB|=\sqrt{(1-5)^2+(2-(-2))^2}$

$=\sqrt{(-4)^2+(2+2)^2}$

$=\sqrt{16+16}=\sqrt{32}$

$=\sqrt{16\times2}=4\sqrt{2}$

$|BC|=\sqrt{(x_3-x_2)^2+(y_3-y_2)^2}$

$=\sqrt{(-2-1)^2+(5-2)^2}$

$=\sqrt{(-3)^2+(3)^2}=\sqrt{9+9}$

$=\sqrt{9\times2}=3\sqrt{2}$

$|AC|=\sqrt{(-2-5)^2+(5-(-2))^2}$

$=\sqrt{(-7)^2+(7)^2}$

$=\sqrt{49+49}$

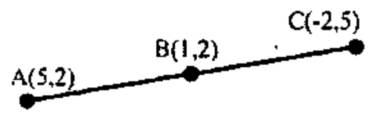
$=\sqrt{49\times2}=7\sqrt{2}$

Since $4\sqrt{2}+3\sqrt{2}=7\sqrt{2}$ is obviously true.

$\therefore\ |AB|+|BC|=|AC|$

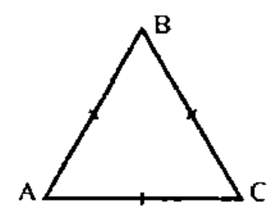
So, the three given points $A=(5,-2)$, $B=(1,2)$ and $C=(-2,5)$ are collinear.

Geometrically, we can arrange these points on a line as shown;



1. Equilateral Triangle:

Definition: That triangle in which all three sides and three angles are equal is called an equilateral triangle. Each angle is of 60°.

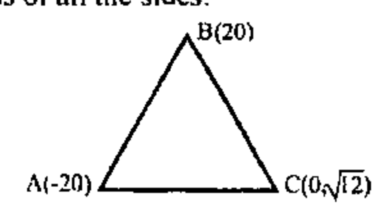


EXAMPLE 7

Prove that the points A(-2,0), B(2,0) and C(0,$\sqrt{12}$) are the vertices of an equilateral triangle.

Solution:

By distance formula, first we find the lengths of all the sides.



$|AB|=\sqrt{(2+2)^2+(0-0)^2}$

$$=\sqrt{(4)^2}=\sqrt{16}=4$$

$$|BC|=\sqrt{(0-2)^2+(\sqrt{12}-0)^2}$$

$$=\sqrt{4+12}=\sqrt{16}=4$$

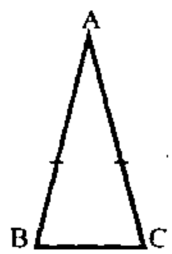
$$|AC|=\sqrt{(0+2)^2+(\sqrt{12}-0)^2}$$

$$=\sqrt{4+12}=\sqrt{16}=4$$

Since $|AB|=|BC|=|AC|=4$, thus the points A, B and C are the vertices of an equilateral triangle.

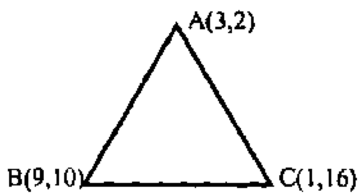
2. Isosceles Triangle:

When the two sides and two angles are equal in a triangle then it is called isosceles triangle. Here $|AB|=|AC|$ and $m\angle B=\angle C$



EXAMPLE 8

Show that the points A(3,2), B(9,10) and C(1,16) are the vertices of an isosceles triangle.



Solution:

Let $|AB|,|BC|$ and $|AC|$ denote the lengths of the side of the triangle. To determine $|AB|,|BC|$ and $|AC|$, distance formula is used.

$$|AB|=\sqrt{(9-3)^2+(10-2)^2}$$

$$=\sqrt{36+64}=\sqrt{100}=10$$

$$|BC|=\sqrt{(1-9)^2+(16-10)^2}$$

$$=\sqrt{64+36}=\sqrt{100}=10$$

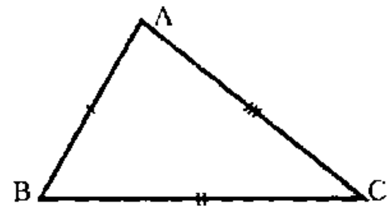
$$|AC|=\sqrt{(1-3)^2+(16-2)^2}$$

$$=\sqrt{4+196}=\sqrt{200}=10\sqrt{2}$$

Since $|AB|=|BC|=10$. thus the two sides AB and AC have equal length so A, B and C are the vertices of an isosceles triangle.

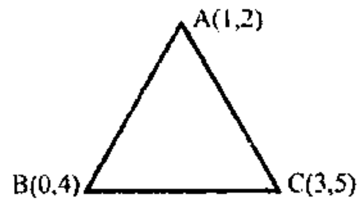
3. Scalene Triangle:

That triangle in which all the three sides and three angles are different is called scalene triangle.



EXAMPLE 9

Show that the points A(1,2), B(0,4) and C(3,5) are the vertices of an scalene triangle.



Solution:

By distance formula, we first find the length of each side.

$$|AB|=\sqrt{(0-1)^2+(4-2)^2}$$

$$=\sqrt{(-1)^2+(2)^2}$$

$$=\sqrt{1+4}=\sqrt{5}$$

$$|BC|=\sqrt{(3-0)^2+(5-4)^2}$$

$$=\sqrt{(3)^2+(1)^2}$$

$$=\sqrt{9+1}=\sqrt{10}$$

$$|AC|=\sqrt{(3-1)^2+(5-2)^2}$$

$$=\sqrt{(2)^2+(3)^2}$$

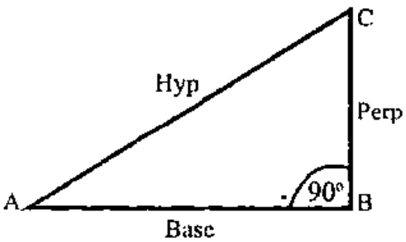
$$=\sqrt{4+9}=\sqrt{13}$$

Since $|AB|\neq|BC|\neq|AC|$ i.e. the lengths of

all the three sides are different. Therefore A, B and C are the vertices of a scalene triangle.

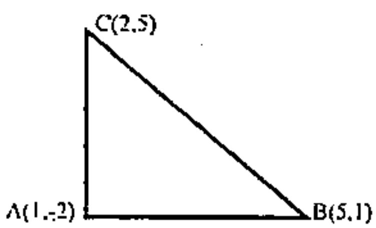
4. Right Angled Triangle:

That triangle in which angle must be of 90° is called right angled triangle. By Pythagoras theorem, *A triangle will be right-angled if* $(Hyp)^2 = (Base)^2 + (Perp)^2$



EXAMPLE 10

Construct the triangle ABC with the help of the points A(1,-2), B(5,1) and C(2,5), and prove that the triangle is a right-angled triangle.



Solution:

Let $|AB|, |BC|$ and $|AC|$ denote the lengths of the sides AB, BC and AC respectively. To determine $|AB|, |BC|$ and $|CA|$, distance formula is used.

$$|AB| = \sqrt{(5-1)^2 + (1+2)^2}$$
$$= \sqrt{(4)^2 + (3)^2}$$
$$= \sqrt{16+9} = \sqrt{25} = 5$$
$$|BC| = \sqrt{(2-5)^2 + (5-1)^2}$$
$$= \sqrt{(-3)^2 + (4)^2}$$
$$= \sqrt{9+16} = \sqrt{25} = 5$$
$$|AC| = \sqrt{(2-1)^2 + (5+2)^2}$$
$$= \sqrt{(1)^2 + (7)^2}$$
$$= \sqrt{1+49} = \sqrt{50} = 5\sqrt{2}$$

Since $(5\sqrt{2})^2 = (5)^2 + (5)^2$

Become $50 = 25 + 25 = 50$

Thus $|AB|^2 + |BC|^2 = |AC|^2$

That is, we have $(Base)^2 + (Perp)^2 = (Hyp)^2$ which is the Pythagoras theorem. Hence the triangle ABC is a right-angled triangle.

EXERCISE 9.2

Q1: Prove that A(-4,-3), B(1,4) and C(6,11) are collinear.

Solution:

Let $(x_1, y_1) = (-4, -3)$, $(x_2, y_2) = (1, 4)$, $(x_3, y_3) = (6, 11)$

Use distance formula

$$|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(1-(-4))^2 + (4-(-3))^2}$$
$$= \sqrt{(1+4)^2 + (4+3)^2}$$
$$= \sqrt{(5)^2 + (7)^2}$$
$$= \sqrt{25+49} = \sqrt{74}$$
$$|BC| = \sqrt{(x_3 - x_2)^2 + (y_3 - y_2)^2}$$
$$= \sqrt{(6-1)^2 + (11-4)^2}$$
$$= \sqrt{(5)^2 + (7)^2}$$
$$= \sqrt{25+49} = \sqrt{74}$$
$$|AC| = \sqrt{(x_3 - x_1)^2 + (y_3 - y_1)^2}$$
$$= \sqrt{(6+4)^2 + (11+3)^2}$$
$$= \sqrt{(10)^2 + (14)^2}$$
$$= \sqrt{100+196}$$
$$= \sqrt{296} = \sqrt{4 \times 74}$$
$$|AC| = 2\sqrt{74}$$

As $\sqrt{74} + \sqrt{74} = 2\sqrt{74}$

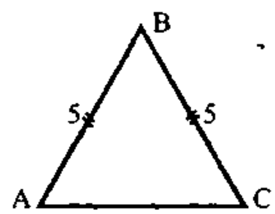
Hence $|AB| + |BC| = |AC|$

Since all points lie on a same straight line, hence the given points A, B and C are collinear.

Q2: Prove that A(-1,3), B(-4,7), C(0,4) is an isosceles triangle.

Solution:

Given $A(x_1,y_1)=(-1,3)$, $B(x_2,y_2)=(-4,7)$ and $C(x_3,y_3)=(0,4)$



Use distance formula

$$|AB|=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$=\sqrt{\left(-4-(-1)\right)^2+(7-3)^2}$$
$$=\sqrt{(-4+1)^2+(7-3)^2}$$
$$=\sqrt{(-3)^2+(4)^2}=\sqrt{9+16}$$
$$=\sqrt{25}=5$$
$$|BC|=\sqrt{(x_3-x_2)^2+(y_3-y_2)^2}$$
$$=\sqrt{\left(0-(-4)\right)^2+(4-7)^2}$$
$$=\sqrt{(0+4)^2+(-3)^2}$$
$$=\sqrt{(4)^2+(-3)^2}$$
$$=\sqrt{16+9}=\sqrt{25}=5$$
$$|AC|=\sqrt{(x_3-x_1)^2+(y_3-y_1)^2}$$
$$=\sqrt{\left(0-(-1)\right)^2+(4-3)^2}$$
$$=\sqrt{(0+1)^2+(1)^2}$$
$$=\sqrt{1+1}=\sqrt{2}$$

As $|AB|=|BC|=5$

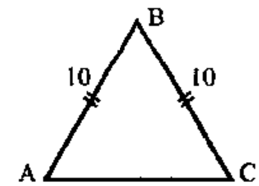
The two sides are equal.

Hence given point A, B, C forms an isosceles triangle.

Q3: Show that the points A(2,3), B(8,11) and C(0,17) are the vertices of an isosceles triangle.

Solution:

Given $A(x_1,y_1)=(2,3)$, $B(x_2,y_2)=(8,11)$ and $C(x_3,y_3)=(0,17)$.



Using distance formula;

$$|AB|=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$=\sqrt{(8-2)^2+(11-3)^2}$$
$$=\sqrt{(6)^2+(8)^2}$$
$$=\sqrt{36+64}$$
$$=\sqrt{100}=10$$
$$|BC|=\sqrt{(x_3-x_2)^2+(y_3-y_2)^2}$$
$$=\sqrt{(0-8)^2+(17-11)^2}$$
$$=\sqrt{(-8)^2+(-6)^2}$$
$$=\sqrt{64+36}$$
$$=\sqrt{100}=10$$

As $|AB|=|BC|$ i.e. the two sides have equal lengths. Hence ΔABC is an isosceles triangle.

Q4: Show that the points A(1,2), B(3,4) and C(0,-1) are the vertices of a scalene triangle. A(-4, -1), B(1,0), C(7,-3).

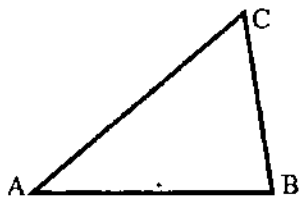
Solution:

Given $A(x_1,y_1)=(1,2)$, $B(x_2,y_2)=(3,4)$ and $C(x_3,y_3)=(0,-1)$

Use distance formula, we get

$$|AB|=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$=\sqrt{(3-1)^2+(4-2)^2}$$
$$=\sqrt{(2)^2+(2)^2}$$

$=\sqrt{4+4}=\sqrt{8}=2\sqrt{2}$



$|BC|=\sqrt{(x_3-x_2)^2+(y_3-y_2)^2}$
$=\sqrt{(0-3)^2+(-1-4)^2}$
$=\sqrt{(-3)^2+(-5)^2}$
$=\sqrt{9+25}=\sqrt{34}$

$|AC|=\sqrt{(x_3-x_1)^2+(y_3-y_1)^2}$
$=\sqrt{(0-1)^2+(-1-2)^2}$
$=\sqrt{(-1)^2+(-3)^2}$
$=\sqrt{1+9}=\sqrt{10}$

Since $|AB|\neq|BC|\neq|AC|$

i.e. All sides are different. Hence ΔABC is a scalene triangle.

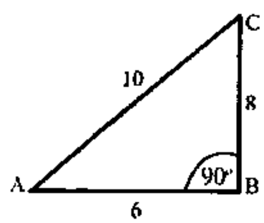
Q5: Prove that points A(-2,-2), B(4,-2) and C(4,6) are vertices of a right triangle.

Solution:

Given $A(x_1,y_1)=(-2,-2)$, $B(x_2,y_2)=(4,-2)$ and $C(x_3,y_3)=(4,6)$

Use the distance formula, we get

$|AB|=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$
$=\sqrt{(4-(-2))^2+(-2-(-2))^2}$
$=\sqrt{(4+2)^2+(-2+2)^2}$
$=\sqrt{(6)^2+0^2}$
$\Rightarrow\sqrt{36}=6$ or $|AB|=6$



$|BC|=\sqrt{(x_3-x_2)^2+(y_3-y_2)^2}$
$=\sqrt{(4-4)^2+(6-(-2))^2}$
$=\sqrt{0^2+(6+2)^2}$
$=\sqrt{(8)^2}=\sqrt{64}=8$

$|AC|=\sqrt{(x_3-x_1)^2+(y_3-y_1)^2}$
$=\sqrt{(4-(-2))^2+(6-(-2))^2}$
$=\sqrt{(4+2)^2+(6+2)^2}$
$=\sqrt{(6)^2+(8)^2}=\sqrt{36+64}$
$\sqrt{100}=10\Rightarrow|AC|=10$

From Pythagoras theorem,
$(Hyp)^2=(Base)^2+(Perp)^2$
$\Rightarrow|AC|^2=|AB|^2+|BC|^2$
$\Rightarrow(10)^2=(6)^2+(8)^2$
$\Rightarrow100=36+64=100$

As Pythagoras theorem is satisfied.
Hence ΔABC is a right triangle.

Q6: Prove that A(-2,0), B(6,0), C(6,6), D(-2,6) are vertices of a rectangle.

Solution:

Given $A(x_1,y_1)=(-2,0)$, $B(x_2,y_2)=(6,0)$, $C(x_3,y_3)=(6,6)$, $D(x_4,y_4)=(-2,6)$

Use the distance formula, we get

$|AB|=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$
$=\sqrt{(6-(-2))^2+(0-0)^2}$
$=\sqrt{(6+2)^2+0}$
$=\sqrt{(8)^2}=8$
$\Rightarrow|AB|=8$

$|CD|=\sqrt{(6-(-2))^2+(6-6)^2}$

$= \sqrt{(6+2)^2 + (0)^2}$

$= \sqrt{(8)^2 + (0)^2} = \sqrt{64+0} = 8$

$\Rightarrow |CD| = 8$

$|BC| = \sqrt{(6-6)^2 + (6-0)^2}$

$= \sqrt{(0)^2 + (6)^2} = \sqrt{36} = 6$

$|AD| = \sqrt{(-2+2)^2 + (6-0)^2}$

$= \sqrt{0 + (6)^2} = 6$

As $|AD| = |CD| = 8$ and $|BC| = |AD| = 6$

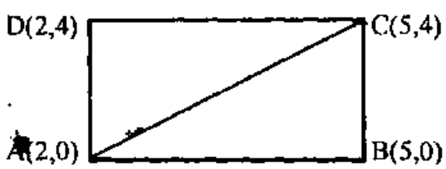
The opposite sides are equal.
Hence $ABCD$ is a rectangle.

Q7: The vertices of the rectangle *ABCD* are A(2,0), B(5,0), C(5,4) and D(2,4). How long is the diagonal AC?

Solution:

Given $A(2,0) = (x_1, y_1)$

$C(5,4) = (x_2, y_2)$



Use the distance formula for diagonal AC

$|AC| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(5-2)^2 + (4-0)^2}$

$= \sqrt{(3)^2 + (4)^2} = \sqrt{9+16}$

$= \sqrt{25} = 5$

Hence length of the diagonal

$|AC| = 5$ Ans.

Q8: Prove that A(-4,-1), B(1,0), C(7,-3) and D(2,-4) are vertices of a parallelogram.

Solution:

Given points are A(-4,-1), B(1,0), C(7,-3), D(2,-4). Use the distance formula,

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \rightarrow (1)$

Put $(x_1, y_1) = (-4, 1)$, $(x_2, y_2) = (1, 0)$ in equation (1)

$|AB| = \sqrt{(1-(-4))^2 + (0-(-1))^2}$

$= \sqrt{(1+4)^2 + (0+1)^2}$

$= \sqrt{(5)^2 + (1)^2}$

$= \sqrt{25+1} = \sqrt{26}$

$d = \sqrt{(x_4 - x_3)^2 + (y_4 - y_3)^2}$

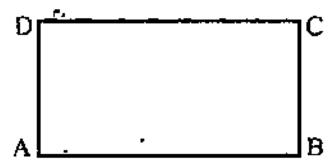
Put $(x_3, y_3) = (7, -3)$ and $(x_4, y_4) = (2, -4)$

$|CD| = \sqrt{(2-7)^2 + (-4-(-3))^2}$

$= \sqrt{(-5)^2 + (-4+3)^2}$

$= \sqrt{(-5)^2 + (-1)^2}$

$= \sqrt{25+1} = \sqrt{26}$



$d = \sqrt{(x_3 - x_2)^2 + (y_3 - y_2)^2}$

Put $(x_2, y_2) = (1, 0)$, $(x_3, y_3) = (7, -3)$

$|BC| = \sqrt{(7-1)^2 + (-3-0)^2}$

$= \sqrt{(6)^2 + (-3)^2}$

$= \sqrt{36+9} = \sqrt{45}$

$d = \sqrt{(x_4 - x_1)^2 + (y_4 - y_1)^2}$

Put $(x_1, y_1) = (-4, -1)$, $(x_4, y_4) = (2, -4)$

$|AD| = \sqrt{(2-(-4))^2 + (-4-(-1))^2}$

$= \sqrt{(2+4)^2 + (-4+1)^2}$

$= \sqrt{(6)^2 + (-3)^2}$

$= \sqrt{36+9} = \sqrt{45}$

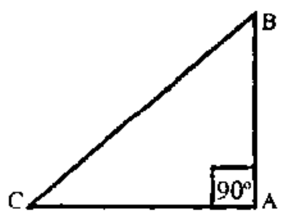
As $|AB| = |CD|$ and $|BC| = |AD|$

The opposite sides are equal.
Hence $ABCD$ is a parallelogram.

Q9: Find *b* such that the points A(2,*b*), B(5,5) and C(-6,0) are the vertices of a right angled triangle with $\angle BAC = 90^\circ$.

Solution:
Given vertices are A(2,b), B(5,5), C(-6,0) then use distance formula,

$$|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(5-2)^2 + (5-b)^2}$$
$$= \sqrt{(3)^2 + (5-b)^2}$$
$$= \sqrt{9 + (5-b)^2}$$



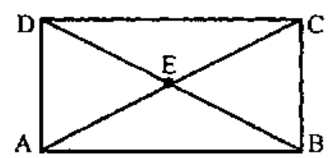
$$|BC| = \sqrt{(-6-5)^2 + (0-5)^2}$$
$$= \sqrt{(-11)^2 + (-5)^2}$$
$$= \sqrt{121+25} = \sqrt{146}$$
$$|AC| = \sqrt{(-6-2)^2 + (0-b)^2}$$
$$= \sqrt{(-8)^2 + b^2} = \sqrt{64+b^2}$$

Given that ΔABC is right triangle.
By Pythagoras theorem,
$(Hypotenuse)^2 = (Base)^2 + (Perp)^2$
$$(BC)^2 = (AC)^2 + (AB)^2$$
$$\left(\sqrt{146}\right)^2 = \left(\sqrt{64+b^2}\right)^2 + \left(\sqrt{9+(5-b)^2}\right)^2$$
$$146 = 64 + b^2 + 9 + (5-b)^2$$
$$146 = 64 + b^2 + 9 + (5)^2 + b^2 - 2(5)(b)$$
$$146 = 64 + 2b^2 + 9 + 25 - 10b$$
Or $0 = 2b^2 - 10b + 98 - 146$
$$\Rightarrow 2b^2 - 10b - 48 = 0$$
Take 2 common
$$2(b^2 - 5b - 24) = 0$$
Divide by 2 both sides
$$\frac{\cancel{2}}{\cancel{2}}\left(b^2 - 5b - 24\right) = \frac{0}{2}$$
$$\Rightarrow b^2 - 5b - 24 = 0$$
$$b^2 - 8b + 3b - 24 = 0$$
$$b(b-8) + 3(b-8) = 0$$
$$\Rightarrow (b-8)(b+3) = 0$$
$$b - 8 = 0 \Rightarrow b = 8$$
Or $b + 3 = 0 \Rightarrow b = -3$

Hence $\boxed{b = 8 \ or \ -3}$ Ans.

Q10: Given A(-4,-2), B(1,-3) and C(3,1), find the coordinate of *D*, in the 2nd quadrant such that quadrilateral *ABCD* is a parallelogram.

Solution:
Given points are $A(-4,-2)$, $B(1,-3)$ and $C(3,1)$
Let $D(x,y)$ be required point
Since *A*, *B*, *C*, *D* are the vertices of parallelogram, then the midpoint of diagonals *AC* and *BD* must be equal.



Let $E(a,b)$ be the midpoint of *AC* and *BD*.
From midpoint formula, we have
$$x = \frac{x_1 + x_2}{2} \text{ and } y = \frac{y_1 + y_2}{2} \rightarrow (1)$$
As $E(a,b)$ is midpoint of *AC*, then
Put $x = a, y = b, \quad x_1 = -4, y_1 = -2, \quad x_2 = 3,$ $y_2 = 1$ in equation (1)
$$a = \frac{-4+3}{2} \text{ and } b = \frac{-2+1}{2}$$
$$\Rightarrow a = -\frac{1}{2} \text{ and } b = -\frac{1}{2}$$
Hence coordinate of midpoint of *AC* and *BD* are
$$E = \left(-\frac{1}{2}, -\frac{1}{2}\right)$$
Since $E = \left(-\frac{1}{2}, -\frac{1}{2}\right)$ is the midpoint of BD.
Where $B(1,-3)$ and $D(x,y)$
Then put in equation (1) again

$$-\frac{1}{2}=\frac{1+x}{2} \quad , \quad -\frac{1}{2}=\frac{-3+y}{2}$$

$$\Rightarrow -1=1+x \quad \text{and} \quad -1=-3+y$$

$$\Rightarrow -1-1=x \quad \text{and} \quad -1+3=y$$

$$\Rightarrow x=-2 \quad \text{and} \quad y=2$$

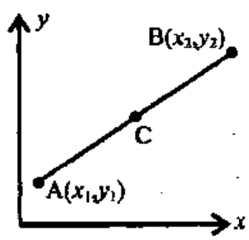
Hence coordinates of $D(x,y)=D(-2,2)$ which lies in 2nd quadrant.

Mid-point Formula:

Let $A(x_1,y_1)$ and $B(x_2,y_2)$ be the two points then the mid-point $C(x,y)$ is given as,

$$C(x,y)=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$

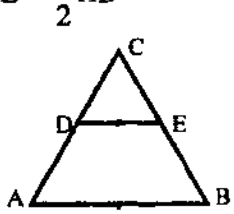
This formula is called the mid-point formula.



Remember:

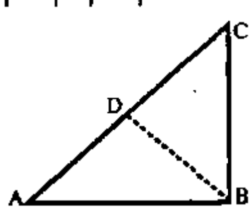
1. A line-segment joining the mid-point of two sides of a triangle is equal to half of the length of 3rd side.

   Here $\overline{DE}=\frac{1}{2}\overline{AB}$

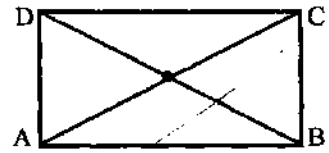


2. The mid-point of the hypotenuse of a right triangle is equidistant from the vertices.

   $|AD|=|BD|=|CD|$

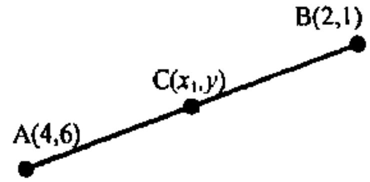


3. The diagonals of a rectangle have equal length. $|AC|=|BD|$



EXAMPLE 14

Find the coordinates of the mid-point of the segment joining the points $A(4,6)$ and $B(2,1)$.



Solution:

Let $C(x,y)$ is the mid-point of $\overline{AB}$ then by the mid-point formula

$$C(x,y)=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$

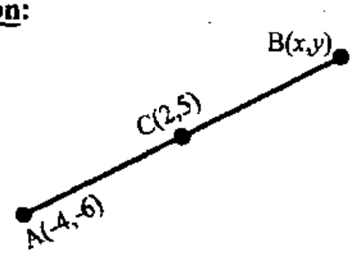
$$=\left(\frac{4+2}{2},\frac{2+6}{2}\right)$$

$$=\left(\frac{6}{2},\frac{8}{2}\right)=(3,4)$$

EXAMPLE 15

The coordinates of mid-point of a line segment AB are (2,5) and that of A are (-4,-6). Find the coordinates of point B.

Solution:



Let the coordinates of B are (x,y) then by the mid-point formula, we can write

$$2=\frac{-4+x}{2} \quad \text{and} \quad 5=\frac{-6+y}{2}$$

$$4=-4+x \quad \text{and} \quad 10=-6+y$$

$x = 8$ and $y = 10 = -6 + y$

$x = 8$ and $y = 16$

- So the coordinates of B are (8, 16).

RESULT 1:

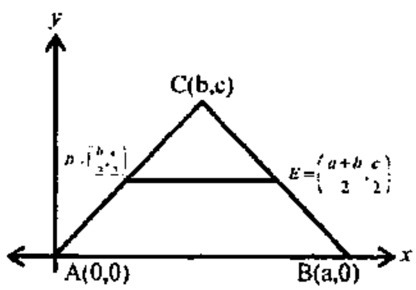
Prove that the line segment joining the mid-points of two sides of a triangle is equal to half of the length of the third side.

Proof:

We choose the axes of coordinates as show and construct a triangle ABC with one vertex at the origin and one side along x-axis, so that

$A = (0,0), B = (a,0)$ and $C = (b,c)$. Let D and E are the mid-points of $\overline{AC}$ and $\overline{BC}$. By mid-points formula D has coordinates.

$$= \left(\frac{b+0}{2}, \frac{c+0}{2}\right)$$



i.e. $D = \left(\frac{b}{2}, \frac{c}{2}\right)$

Similarly $E = \left(\frac{a+b}{2}, \frac{c+0}{2}\right)$ or $E = \left(\frac{a+b}{2}, \frac{c}{2}\right)$

By distance formula

$$|DE| = \sqrt{\left(\frac{a+b}{2} - \frac{b}{2}\right)^2 + \left(\frac{c}{2} - \frac{c}{2}\right)^2}$$

$$= \sqrt{\left(\frac{a+b-b}{2}\right)^2 + 0} = \sqrt{\left(\frac{a}{2}\right)^2}$$

i.e. $|DE| = \frac{a}{2} \longrightarrow (1)$

again $|AB| = \sqrt{(a-0)^2 + (0+0)^2}$

$= \sqrt{a^2} = a$

i.e. $|AB| = a \longrightarrow (2)$

From (1) and (2)

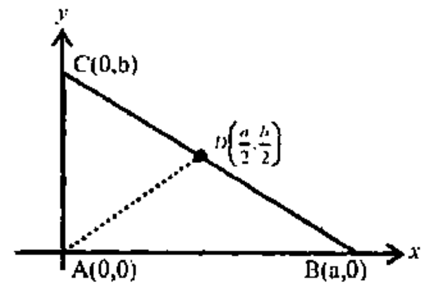
$$|DE| = \frac{a}{2} = \frac{1}{2}a = \frac{1}{2}|AB|$$

i.e. $|DE| = \frac{1}{2}|AB|$

This proves the required result.

RESULT 2:

The mid-point of the hypotenuse of a right triangle is equidistant from the vertices.



Proof:

The triangle ABC is a right triangle. $\overline{BC}$ is the hypotenuse and D is its mid-point. The coordinates of the vertices are:

$A = (0,0)$, $B = (a,0)$, $C = (0,b)$

Coordinates of $D = \left(\frac{a+0}{2}, \frac{0+b}{2}\right)$ by mid-point formula

Coordinates of $D = \left(\frac{a}{2}, \frac{b}{2}\right)$

We are to prove that $|AD| = |BD| = |CD|$.

Using distance formula, we have

$$|AD| = \sqrt{\left(\frac{a}{2} - 0\right)^2 + \left(\frac{b}{2} - 0\right)^2}$$

$$= \sqrt{\frac{a^2}{4} + \frac{b^2}{4}} = \sqrt{\frac{a^2 + b^2}{4}} \longrightarrow (1)$$

$$|BD| = \sqrt{\left(a - \frac{a}{2}\right)^2 + \left(0 - \frac{b}{2}\right)^2}$$

$$= \sqrt{\left(\frac{2a-a}{2}\right)^2 + \left(-\frac{b}{2}\right)^2}$$

$$=\sqrt{\left(\frac{a}{2}\right)^2+\left(-\frac{b}{2}\right)^2}=\sqrt{\frac{a^2}{4}+\frac{b^2}{4}}$$

$$=\sqrt{\frac{a^2+b^2}{4}}\longrightarrow(2)$$

$$|CD|=\sqrt{\left(\frac{a}{2}-0\right)^2+\left(\frac{b}{2}-b\right)^2}$$

$$=\sqrt{\frac{a^2}{4}+\left(\frac{b-2b}{2}\right)^2}$$

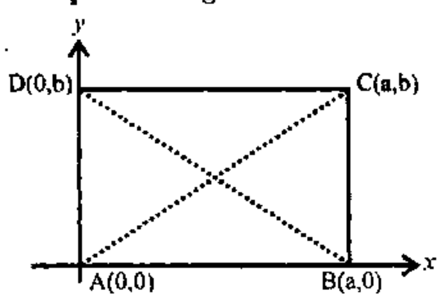
$$|CD|=\sqrt{\frac{a^2}{4}+\left(-\frac{b}{2}\right)^2}$$

$$=\sqrt{\frac{a^2}{4}+\frac{b^2}{4}}=\sqrt{\frac{a^2+b^2}{4}}\longrightarrow(3)$$

Comparing (1), (2) and (3) we have $|AD|=|BD|=|CD|$ proved.

RESULT 3:

Verify that the diagonals of any rectangle are equal in length.



Proof:

In the figure we have shown a rectangle $ABCD$ with coordinates of its vertices. $\overline{AC}$ and $\overline{BD}$ are the diagonals. We are to prove that $|AC|=|BD|$. Using distance formula,

$$|AC|=\sqrt{(a-0)^2+(b-0)^2}$$

$$=\sqrt{a^2+b^2}\longrightarrow(1)$$

$$|BD|=\sqrt{(a-0)^2+(0-b)^2}$$

$$=\sqrt{a^2+b^2}\longrightarrow(2)$$

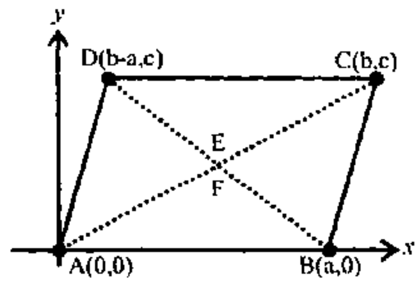
From (1) and (2)

$|AC|=|BD|$

This proves the required result.

RESULT 4:

Show that diagonals of a parallelogram bisect each other.



Proof:

Let $ABCD$ is a parallelogram whose vertices are shown i.e.

$A=(0,0)$, $B=(a,0)$

$C=(b,c)$, $D=(b-a,c)$.

Let the mid-point of the diagonal $\overline{AC}$ is E then coordinates of

$$E=\left(\frac{b+0}{2},\frac{c+0}{2}\right)=\left(\frac{b}{2},\frac{c}{2}\right)$$

By mid-point formula

Let the mid-point of the diagonal $\overline{BD}$ is F then coordinates of

$$F=\left(\frac{a+b-a}{2},\frac{0+c}{2}\right)=\left(\frac{b}{2},\frac{c}{2}\right)$$

It means that E and F are same.

$\therefore$ $|AE|=|EC|$ and $|BF|=|FC|$

$\therefore$ Diagonals of a parallelogram bisect each other.

EXERCISE 9.3

Q1: Find the coordinates of the mid-point of the segment with the given endpoints.

i) **(8,-5) and (-2,9)**
ii) **(7,6) and (3,2)**
iii) **(-2,3) and (-9,-6)**
iv) **($a+b$, $a-b$) and ($-a$, b)**

Solution:

i) (8,-5) and (-2,9)

Given points (8,-5) and (-2,9)

Let $(x_1,y_1)=(8,-5)$ and $(x_2,y_2)=(-2,9)$

Then mid-point formula is,

$$(x,y)=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)\longrightarrow(1)$$

Put values in equation (1)

$$=\left(\frac{8-2}{2},\frac{-5+9}{2}\right)=\left(\frac{6}{2},\frac{4}{2}\right)=(3,2)\text{ Ans.}$$

ii) (7,6) and (3,2)

Let $(x_1,y_1)=(7,6)$ and $(x_2,y_2)=(3,2)$

Then mid-point formula is,

$$(x,y)=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)\to(1)$$

Put values in equation (1)

$$=\left(\frac{7+3}{2},\frac{6+2}{2}\right)$$

$$=\left(\frac{10}{2},\frac{8}{2}\right)=(5,4)\text{ Ans.}$$

iii) (-2,3) and (-9,-6)

Let $(x_1,y_1)=(-2,3)$ and $(x_2,y_2)=(-9,-6)$

Then mid-point formula is,

$$(x,y)=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)\to(1)$$

Put values in equation (1)

$$=\left(\frac{-2-9}{2},\frac{3-6}{2}\right)$$

$$=\left(\frac{-11}{2},\frac{-3}{2}\right)\text{ Ans.}$$

iv) ($a+b$, $a-b$) and ($-a$, b)

Let $(x_1,y_1)=(a+b,a-b)$ and $(x_2,y_2)=(-a,b)$

Then mid-point formula is,

$$(x,y)=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)\to(1)$$

Put values in equation (1)

$$=\left(\frac{a+b-a}{2},\frac{a-b+b}{2}\right)$$

$$=\left(\frac{\cancel{a}+b-\cancel{a}}{2},\frac{a-\cancel{b}+\cancel{b}}{2}\right)$$

$$=\left(\frac{b}{2},\frac{a}{2}\right)\quad\text{Ans.}$$

Q2: The mid-point and one end of a lien segment are (3,7) and (4,2) respectively. Find the other end point.

Solution:

Given mid-point $(x,y)=(3,7)$ and one end $(x_1,y_1)=(4,2)$

We find the other end (x_2,y_2)

Using mid-point formula

$$(x,y)=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)\to(1)$$

Put values in equation (1)

$$(3,7)=\left(\frac{4+x_2}{2},\frac{2+y_2}{2}\right)$$

$$\Rightarrow\frac{4+x_2}{2}=3$$

$$4+x_2=6$$

$$\Rightarrow x_2=6-4=2$$

And $\frac{2+y_2}{2}=7$

$$\Rightarrow 2+y_2=14$$

$$\Rightarrow y_2=14-2=12$$

$\therefore$ The other end point $=(2,12)$ Ans.

Q3: The midpoints of the sides of a triangle are (2,5), (4,2), (1,1). Find the coordinates of the three vertices.

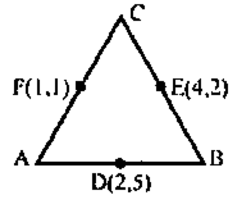
Solution:

Let $A(x_1,y_1), B(x_2,y_2)$ and $C(x_3,y_3)$ be the required vertices.

Then from midpoint formula,

$$(2,5)=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$

Comparing the corresponding elements on both sides



Now $\frac{x_1+x_2}{2}=2$

$$\Rightarrow x_1+x_2=4\to(1)$$

$\frac{y_1+y_2}{2}=5$

$\Rightarrow y_1+y_2=10 \to (2)$

By midpoint formula,

$(1,1)=\left(\frac{x_1+x_3}{2}, \frac{y_1+y_3}{2}\right)$

Now $\frac{x_1+x_3}{2}=1$

$\Rightarrow x_1+x_3=2 \to (3)$

$\frac{y_1+y_3}{2}=1$

$\Rightarrow y_1+y_3=2 \to (4)$

Use midpoint formula again,

$(4,2)=\left(\frac{x_2+x_3}{2}, \frac{y_2+y_3}{2}\right)$

$\frac{x_2+x_3}{2}=4$

$\Rightarrow x_2+x_3=8 \to (5)$

$\frac{y_2+y_3}{2}=2$

$\Rightarrow y_2+y_3=4 \to (6)$

Adding (1), (3), (5) we get

$x_1+x_2+x_1+x_3+x_2+x_3=4+2+8$

$2x_1+2x_2+2x_3=14$

$\Rightarrow 2(x_1+x_2+x_3)=14$

Divide by 2

$\Rightarrow x_1+x_2+x_3=7 \to (7)$

Putting equation (5) in equation (7)

$x_1+8=7$

$\Rightarrow x_1=7-8=-1$

Put equation (3) in equation (7)

$2+x_2=7$

$\Rightarrow x_1=7-2=5$

Put equation (1) in equation (7)

$7+x_3=7 \Rightarrow x_3=7-7=0$

Adding equation (2), (4), (6)

$y_1+y_2+y_2+y_3+y_3+y_1=10+2+4$

$2y_1+2y_2+2y_3=14$

$\Rightarrow 2(y_1+y_2+y_3)=16$

Divide by 2

$\Rightarrow y_1+y_2+y_3=8 \to (8)$

Put equation (2) in equation (8)

$10+y_3=8$

$\Rightarrow y_3=8-10=-2$

Put equation (4) in equation (8)

$2+y_2=8$

$\Rightarrow y_2=8-2=6$

Put equation (6) in equation (8)

$4+y_1=8$

$\Rightarrow y_1=8-4=4$

$\therefore A(x_1,y_1)=A(-1,4), B(x_2,y_2)=B(5,6), C(x_3,y_3)=C(0,-2)$ are the required coordinates.

Q4: The distance between two points with coordinates (1,1) and (4,y) is 5.

Solution:

From distance formula, we know

$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2} \to (1)$

Putting values in equation (1)

$5=\sqrt{(4-1)^2+(y-1)^2}$

OR $\sqrt{9+(y-1)^2}=5$

Squaring both sides

$\sqrt{9+(y-1)^2}=(5)^2$

$9+(y-1)^2=25$

OR $(y-1)^2+9=25$

OR $y^2-2y+1+9=25$

$\Rightarrow y^2-2y+10=25$

$\Rightarrow y^2-2y+10-25=0$

$\Rightarrow y^2-2y-15=0$

By factorization

$\Rightarrow y^2-5y+3y-15=0$

$\Rightarrow y(y-5)+3(y-5)=0$

$\Rightarrow (y-5)(y+3)=0$

$\Rightarrow y-5=0$ or $y+3=0$

$\Rightarrow y=5$ or $y=-3$

Hence $y=\boxed{5,-3}$ Ans.

Review Exercise 9

Q1: Select the correct answer.

i) The point having its coordinates (0,0) is called:

(a) Abscissa (b) Ordinate

✓**(c) Origin** (d) Critical point

ii) The point (-3, 4) lies in which quadrant?

(a) I ✓**(b) II**

(c) III (d) IV

iii) A triangle having all the three sides equal is:

(a) Scalene

(b) Isosceles

✓**(c) Equilateral**

(d) None of these

iv) The only point lying on both the axes is:

(a) (1, 0) (b) (0, 1)

(c) (-1, 0) ✓**(d) (0, 0)**

v) Find the midpoint of a segment with endpoints at (5, 1) & (1, -3).

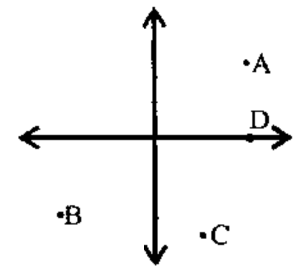
(a) (-1, 3) (b) (-2, 3)

(c) (-2, 2) ✓**(d) (3, -1)**

vi) What are the coordinates of point (2,-3) after it is reflected over the x-axis?

✓**(a) (2, 3)** (b) (-2, 3)

(c) (-2, -3) (d) (2, -3)



vii) Which of the following are the coordinates of point A?

(a) (4, 5) ✓**(b) (5, 4)**

(c) (5, -4) (d) (4, -5)

viii) What is the distance between points B and D?

(a) 5 (b) 11

(c) 12 ✓**(d) 13**

ix) Find the coordinates of the midpoint of the line joining A and C.

✓**(a) (4, -1)** (b) (-1, 4)

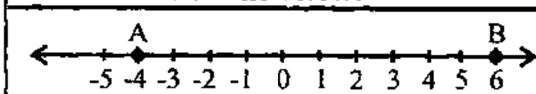
(c) (5, -2) (d) (-2, 5)

x) If A(3, 0) and B(0, 3) are any two points in a plane, then $|AB|=?$

(a) 18 ✓**(b) $\sqrt{18}$**

(c) $9\sqrt{2}$ (d) Zero

Q2: Find the distance between *A* and *B* on the number line below:



Solution:

$|AB|=|b-a|=|6-(-4)|$

Where $b=6, a=-4$

$\Rightarrow |6+4|=|10|=10$ Ans.

Q3: What is the distance between two points with coordinates of (1,-5) and (-5,7)?

Solution:

Given $A(1,-5), B(-5,7)$

Here $x_1 = 1$, $y_1 = -5$, $x_2 = -5$, $y_2 = 7$

From distance formula,

$$|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \rightarrow (1)$$

Putting the values in equation (1)

$$= \sqrt{(-5-1)^2 + (7-(-5))^2}$$

$$= \sqrt{(-6)^2 + (7+5)^2}$$

$$= \sqrt{36 + (12)^2}$$

$$= \sqrt{36+144} = \sqrt{180}$$

$$= \sqrt{4 \times 9 \times 5} = 6\sqrt{5} \quad \text{Ans.}$$

Q4: Using distance formula, show that the points (4,-3), (2,0), (-2,6) are collinear.

Solution:

Given $A(4,-3), B(2,0), C(-2,6)$

Here $x_1 = 4$, $y_1 = -3$, $x_2 = 2$, $y_2 = 0$, $x_3 = -2$, $y_3 = 6$

Use distance formula,

$$|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$|AB| = \sqrt{(2-4)^2 + (0+3)^2}$$

$$= \sqrt{(-2)^2 + (3)^2}$$

$$= \sqrt{4+9} = \sqrt{13} \rightarrow (1)$$

$$|BC| = \sqrt{(x_3 - x_2)^2 + (y_3 - y_2)^2}$$

$$|BC| = \sqrt{(-2-2)^2 + (6-0)^2}$$

$$= \sqrt{(-4)^2 + (6)^2}$$

$$= \sqrt{16+36} = \sqrt{52}$$

$$= \sqrt{13 \times 4} = 2\sqrt{13} \rightarrow (2)$$

$$|AC| = \sqrt{(x_3 - x_1)^2 + (y_3 - y_1)^2}$$

$$|AC| = \sqrt{(-2-4)^2 + (6+3)^2}$$

$$= \sqrt{(-6)^2 + (9)^2}$$

$$= \sqrt{36+81} = \sqrt{117}$$

$$= \sqrt{9 \times 13} = 3\sqrt{13} \rightarrow (3)$$

From equation (1), (2) and (3) we have,

$$|AB| + |BC| = |AC|$$

$$\Rightarrow \sqrt{13} + 2\sqrt{13} = 3\sqrt{13}$$

$$3\sqrt{13} = 3\sqrt{13}$$

Hence A, B, C are collinear.

Q5: Find the point on the x-axis which is equidistant from (0,1) and (3,3).

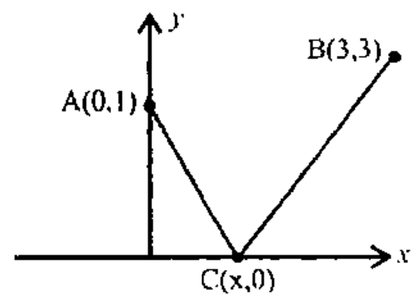
Solution:

Given points are A(0,1), B(3,3)

Let C(x,0) be the required point on x-axis

From given condition;

$$|AC| = |BC|$$



Use distance formula;

$$\sqrt{(x-x_1)^2 + (y-y_1)^2} = \sqrt{(x-x_2)^2 + (y-y_2)^2}$$

Where $(x_1, y_1) = (0,1)$ and $(x_2, y_2) = (3,3)$

$$\sqrt{(x-0)^2 + (0-1)^2} = \sqrt{(x-3)^2 + (0-3)^2}$$

$$\Rightarrow \sqrt{x^2+1} = \sqrt{(x-3)^2 + 9}$$

Squaring both sides, we get'

$$x^2 + 1 = (x-3)^2 + 9$$

$$\cancel{x^2} + 1 = \cancel{x^2} + 9 - 6x + 9$$

$$\Rightarrow 1 = 18 - 6x \Rightarrow 6x = 18 - 1$$

$$\Rightarrow 6x = 17 \Rightarrow x = \frac{17}{6}$$

Hence required point $C\left(\frac{17}{6}, 0\right)$ Ans.

Q6: A segment has one endpoint at (15,22) and a midpoint at (5,18), what are the coordinates of the other end-point?

Solution:

Let $A(15,22)$ be one end point and $B(x,y)$ be the other endpoint. Since $C(5,18)$ be the midpoint of AB. Then by midpoint formula,

$$5=\frac{15+x}{2} \quad \text{and} \quad 18=\frac{22+y}{2}$$

$$\Rightarrow 10=15+x \quad \text{and} \quad 36=22+y$$

$$\Rightarrow 10-15=x \quad \text{and} \quad 36-22=y$$

$$\Rightarrow y=-5 \quad \text{and} \quad y=14$$

Hence coordinate of other endpoint is $B(-5,14)$ Ans.

Q7: Prove that (2,1), (0,0), (-1,2) and (1,3) are the vertices of a rectangle.

Solution:

Given points are A(2,1), B(0,0), C(-1,2) and D(1,3)

Use distance formula,

$$d=\sqrt{(x_2-x_1)+(y_2-y_1)^2} \rightarrow (1)$$

For length of side AB

Put $x_1=2,\ y_1=1,\ x_2=0,\ y_2=0$ in eq. (1)

$$|AB|=\sqrt{(0-2)^2+(0-1)^2}$$

$$=\sqrt{(-2)^2+(-1)^2}$$

$$=\sqrt{4+1}=\sqrt{5}$$

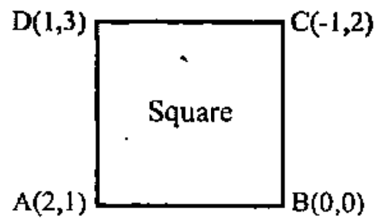
For length of side BC,

Put $x_1=0,\ y_1=0,\ x_2=-1,\ y_2=2$ in eq. (2)

$$|BC|=\sqrt{(-1-0)^2+(2-0)^2}$$

$$=\sqrt{(-1)^2+(2)^2}$$

$$\Rightarrow |BC|=\sqrt{1+4}=\sqrt{5}$$



For length of side DC

Put $x_1=1,\ y_1=3,\ x_2=-1,\ y_2=2$ in eq. (1)

$$\Rightarrow |DC|=\sqrt{(-1-1)^2+(2-3)^2}$$

$$=\sqrt{(-2)^2+(-1)^2}$$

$$=\sqrt{4+1}=\sqrt{5}$$

For length of side AD

Put $x_1=2,\ y_1=1,\ x_2=1,\ y_2=3$ in eq. (1)

$$\Rightarrow |AD|=\sqrt{(1-2)^2+(3-1)^2}$$

$$=\sqrt{(-1)^2+(2)^2}$$

$$=\sqrt{1+4}=\sqrt{5}$$

As $|AB|=|BC||CD|=|AD|$

Hence A, B, C, D are the vertices of a square.

NOTE: Every square is also a rectangle.

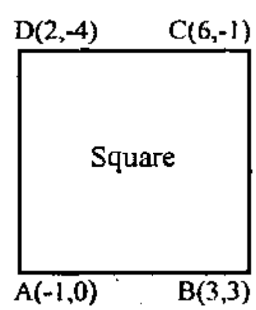
Q8: Prove that A(-1,0), B(3,3), C(6,-1) and D(2,-4) is a square.

Solution:

Given points are A(-1,0), B(3,3), C(6,-1), D(2,-4)

Use distance formula,

$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2} \rightarrow (1)$$



For length of side AB

Put $x_1=-1,\ y_1=0,\ x_2=3,\ y_2=3$ in eq. (1)

$\Rightarrow |AB|=\sqrt{(3-(-1))^2+(3-0)^2}$

$=\sqrt{(3+1)^2+(3)^2}$

$=\sqrt{(4)^2+9}=\sqrt{16+9}=\sqrt{25}$

$\Rightarrow |AB|=5$

For side BC,

Put $x_1=3,\ y_1=3,\ x_2=6,\ y_2=-1$ in (1)

$|BC|=\sqrt{(6-3)^2+(-1-3)^2}$

$=\sqrt{(3)^2+(-4)^2}$

$\Rightarrow |BC|=\sqrt{9+16}=\sqrt{25}=5$

For length of side DC,

Put $x_1=2, y_1=-4,\ x_2=6,\ y_2=-1$ in (1)

$\Rightarrow |DC|=\sqrt{(6-2)^2+(-1-(-4))^2}$

$=\sqrt{(4)^2+(-1+4)^2}$

$=\sqrt{16+(3)^2}=\sqrt{16+9}$

$=\sqrt{25}=5$

For length of side AD,

Put $x_1=-1,\ y_1=0,\ x_2=2,\ y_2=-4$ in (1)

$\Rightarrow |AD|=\sqrt{(2-(-1))^2+(-4-0)^2}$

$=\sqrt{(2+1)^2+(-4)^2}$

$=\sqrt{(3)^2+16}=\sqrt{9+16}$

$=\sqrt{25}=5$

As $|AB|=|BC|=|CD|=|AD|$

Hence A, B, C and D are the vertices of a square.

Q9: Show that the triangle with the vertices (6,5), (2,-4) and (5,-1) is an isosceles triangle.

Solution:

Given vertices are A(6,5), B(2,-4), C(5,-1)

Use distance formula,

$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

For length of side AB,

Put $x_1=6,\ y_1=5,\ x_2=2,\ y_2=-4$ in (1)

$\Rightarrow |AB|=\sqrt{(2-6)^2+(-4-5)^2}$

$=\sqrt{(-4)^2+(-9)^2}$

$=\sqrt{16+81}=\sqrt{97}$

$\Rightarrow |AB|=\sqrt{97}$

For side BC,

Put $x_1=2,\ y_1=-4,\ x_2=5,\ y_2=-1$ in (1)

$|BC|=\sqrt{(5-2)^2+(-1-(-4))^2}$

$=\sqrt{(3)^2+(-1+4)^2}$

$\Rightarrow |BC|=\sqrt{9+(3)^2}$

$=\sqrt{9+9}=\sqrt{18}$

$=\sqrt{9\times 2}=\sqrt{9}\times\sqrt{2}=3\sqrt{2}$

For length of side AC,

Put $x_1=6,\ y_1=5,\ x_2=5,\ y_2=-1$ in (1)

$\Rightarrow |AC|=\sqrt{(5-6)^2+(-1-5)^2}$

$=\sqrt{(-1)^2+(-6)^2}$

$=\sqrt{1+36}=\sqrt{37}$

Since $|AB|\neq|BC|\neq|AC|$

i.e. $\sqrt{97}\neq 3\sqrt{2}\neq\sqrt{37}$

Hence ABC is not an isosceles triangle, which is a scalene triangle.

***

Additional MCQs of Unit 9:
Introduction to Coordinate Geometry

1. The study of figures and their equations is called..........geometry.
   (a) Analytic (b) Practical (c) Coordinate (d) none
   ✓ **Ans. (c) Coordinate**

2. $d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ is the..........formula.
   (a) Midpoint (b) Distance (c) Ratio (d) Length
   ✓ **Ans. (b) Distance**

3. The distance can never be..........
   (a) Zero (b) Positive (c) Negative (d) none
   ✓ **Ans. (c) Negative**

4. $d=|x_2-x_1|$ shows..........distance.
   (a) Vertical (b) Horizontal (c) Directed (d) none
   ✓ **Ans. (b) Horizontal**

5. The distance between (2,3) and (7,3) is..........
   (a) 6 (b) 2 (c) 8 (d) 5
   ✓ **Ans. (d) 5**

6. If the three points lie on same line then they are..........
   (a) Collinear (b) Coplanar (c) Vertical (d) none
   ✓ **Ans. (a) Collinear**

7. If all the three sides of a triangle are equal then it is..........triangle.
   (a) Isosceles (b) Equilateral (c) Scalene (d) none
   ✓ **Ans. (b) Equilateral**

8. $(x,y)=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$ is.. ....formula.
   (a) Distance (b) Ratio (c) Slope (d) Midpoint
   ✓ **Ans. (d) Midpoint**

9. The midpoint of A(4,6) and B(2,1) is..........
   (a) (-3, -4) (b) (-3, 4) (c) (3,4) (d) (5,4)
   ✓ **Ans. (c) (3, 4)**

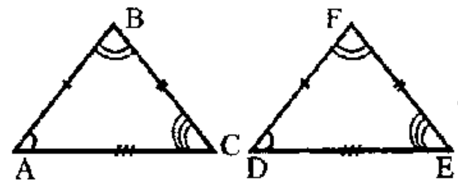
10. A triangle having one angle of 90° is..........triangle.
    (a) Right (b) Isosceles (c) Equilateral (d) none
    ✓ **Ans. (a) Right**

UNIT 10:
CONGRUENT TRIANGLES

Two plane figures are congruent if they have same size and same shape. A triangle has six elements. In which three are sides and three are angles.

Definition:

Two triangles ΔABC and ΔDEF are said to be congruent if the following six conditions are hold:



i) $\angle A \cong \angle D$
ii) $\angle B \cong \angle E$
iii) $\angle C \cong F$
iv) $\overline{AB} \cong \overline{DF}$
v) $\overline{BC} \cong \overline{EF}$
vi) $\overline{AC} \cong \overline{DE}$

Then we write $\Delta ABC \cong \Delta DEF$

Properties of Congruent Triangles:

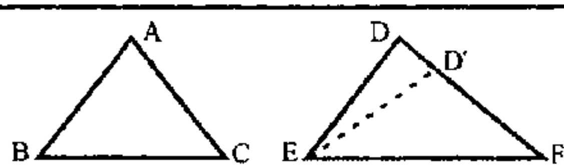
1. $\Delta ABC \cong \Delta ABC$, this is called identity congruence which means every triangle is congruent to itself.
2. If $\Delta ABC \cong \Delta DEF$ then $\Delta DEF \cong \Delta ABC$, this is called symmetric property of congruence.
3. If $\Delta ABC \cong \Delta DEF$ and $\Delta DEF \cong \Delta XYZ$ then $\Delta ABC \cong \Delta XYZ$, this is called transitive property of congruence.

Remember: $\boxed{S.A.S}$

If two sides and their included angle of one triangle are congruent to two sides and their included angle of another triangle then the two triangles are congruent.

THEOREM 10.1

Statement: If two angles and a non-included side of one triangle are congruent to the corresponding angles and non-included side of another triangle, then the triangles are congruent.



Given: In $\Delta ABC \longleftrightarrow \Delta DEF$

$\overline{BC} \cong \overline{EF}$, $\angle B \cong \angle E$ and $\angle C \cong \angle F$

To Prove: $\Delta ABC \cong \Delta DEF$

Construction: Suppose $\overline{AC} \not\cong \overline{DF}$ and $\overline{AC} \cong \overline{D'F}$, where D' is a point on $\overline{DF}$. Join D' to E.

Proof:

Statements	Reasons
In $\Delta ABC \longleftrightarrow \Delta D'EF$	

$\overline{BC} \cong \overline{EF}$	Given
$\angle C \cong \angle F$	Given
$\overline{AC} \cong \overline{D'F}$	Supposition
$\therefore \Delta ABC \cong \Delta D'EF$	(S.A.S ≅ S.A.S)
Hence $\angle B \cong \angle D'EF$	Corresponding angles of congruent triangles
But $\angle DEF \cong \angle B$	Given
$\therefore \angle D'EF \cong DEF$	Transitive property
$\Rightarrow \overline{ED} \cong \overline{ED'}$	
Thus $\overline{AC} \cong \overline{DF}$	
In $\Delta ABC \longleftrightarrow \Delta DEF$	
$\overline{BC} \cong \overline{EF}$	Given
$\overline{AC} \cong \overline{DF}$	Already proved
$\angle C \cong \angle F$	Given
$\therefore \Delta ABC \cong \Delta DEF$	(S.A.S) ≅ (S.A.S)

THEOREM 10.2

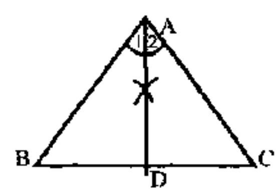
Statement: If two angles of a triangle are congruent then the side opposite to these angles must be congruent.

Given: In $\Delta ABC, \angle B \cong \angle C$

To Prove: $\overline{AB} \cong \overline{AC}$

Construction:

Draw bisector of $\angle A$ cutting $\overline{BC}$ at point D.



Proof:

Statements	Reasons
In $\Delta ABC \longleftrightarrow \Delta ACD$	
$\overline{AD} \cong \overline{AD}$	Common
$m\angle 1 \cong m\angle 2$	Construction
$m\angle B \cong m\angle C$	Given
$\therefore \Delta ABD \cong \Delta ACD$	(A.A.S ≅ A.A.S)
Hence $\overline{AB} \cong \overline{AC}$	Corresponding sides of congruent triangles

THEOREM 10.3

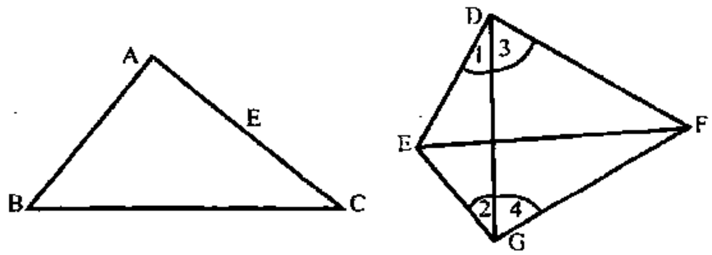
Statement: In a correspondence of two triangle, if three sides of one triangle are congruent to the corresponding three sides of the other, the two triangles are congruent.

Given: In $\Delta ABC \longleftrightarrow \Delta DEF$

$\overline{AB} \cong \overline{DE}, \overline{BC} \cong \overline{EF}$ and $\overline{CA} \cong \overline{FD}$

To Prove: $\Delta ABC \cong \Delta DEF$

Construction: Suppose $\overline{BC}$ is not shorter than any of the other two sides of ΔABC. Construct ΔGEF such that $\overline{GE} \cong \overline{AB}$. Join G with D.



Proof:

Statements	Reasons
In $\Delta ABC \longleftrightarrow \Delta GEF$	
$\overline{BC} \cong \overline{EF}$	Given
$\therefore m\angle B \cong \angle GEF$	Construction
$\overline{AB} \cong \overline{GE}$	Construction
$\therefore \Delta ABC \cong \Delta GEF$	(S.A.S ≅ S.A.S)
So $\overline{GF} \cong \overline{AC}$	Corresponding sides of congruent triangles
But $\overline{DF} \cong \overline{AC}$	Given
$\therefore \overline{GF} \cong \overline{DF} \longrightarrow (1)$	Transitive property
In ΔFDG,	
$\angle 3 \cong \angle 4 \longrightarrow (2)$	ΔDFG is isosceles
In ΔEGD,	
$\overline{DE} \cong \overline{EG}$	Each is congruent to $\overline{AB}$
$\therefore \angle 1 \cong \angle 2 \longrightarrow (3)$	By (2) and (3)
$\therefore \angle 3 + \angle 1 \cong \angle 4 + \angle 2$	
$\therefore m\angle D \cong m\angle G$	Addition of angles postulate
In the correspondence	
In $\Delta DEF \longleftrightarrow \Delta GEF$	
$\overline{DE} \cong \overline{GE}$	Proved
$\angle D \cong \angle G$	Proved
$\overline{DE} \cong \overline{FG}$	From equation (1)
$\therefore \Delta DEF \cong \Delta GEF$	(S.A.S)
But $\Delta ABC \cong \Delta GEF$	Proved
Hence $\Delta ABC \cong \Delta GEF$	From transitive property

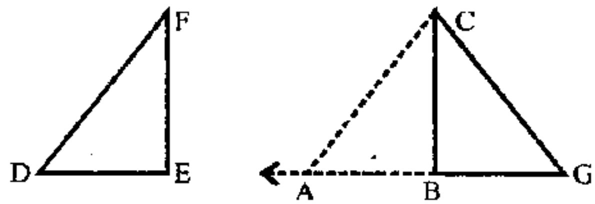
THEOREM 10.4

Statement: If in the correspondence of two triangles, the hypotenuse and one side of one are congruent to the hypotenuse and one side of other, then triangles are congruent.

Given: In $\Delta ABC, \Delta DEF$

$\angle ABC \cong \angle E$ (Each is right angle)

$\overline{AC} \cong \overline{DF}$ and $\overline{BC} \cong \overline{EF}$



To Prove: $\Delta ABC \cong \Delta DEF$

Construction:

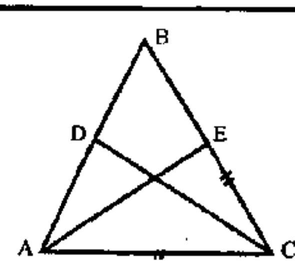
Produce $\overline{AB}$ to the point G such that $\overline{BG} \cong \overline{DE}$. Join C with G.

Proof:

Statements	Reasons
In ΔGBC	
$m\angle CBG = 90^\circ$	Given
In $\Delta GBC \longleftrightarrow \Delta DEF$	
$\overline{GB} \cong \overline{DE}$	Construction
$\angle GBC \cong \angle DEF$	Each is right angle or 90°
$\therefore \Delta GBC \cong \Delta DEF$	(S.A.S ≅ S.A.S)
$\overline{GC} \cong \overline{DF}$	Correspondence sides of congruent triangles
But $\overline{DF} \cong \overline{AC}$	Given
$\therefore \overline{GC} \cong \overline{AC}$	
$\Rightarrow \angle A \cong \angle G$	Angles opposite to congruent sides of ΔCAG
In $\Delta ABC \longleftrightarrow \Delta GBC$	
$\overline{AC} \cong \overline{GC}$	Proved
$\angle A \cong \angle G$	Proved
$\angle ABC \cong \angle GBC$	Each is of 90°
$\therefore \Delta ABC \cong \Delta GBC$	(A.A.S)
But $\Delta GBC \cong \Delta DEF$	Proved
$\therefore \Delta ABC \cong \Delta DEF$	From transitive property

EXERCISE 10.1

Q1: Prove that the perpendiculars drawn from the endpoints of the base of an isosceles triangle to their opposite sides are congruent.



Given: ΔABC in which $\overline{AC} \cong \overline{BC}$

And $\overline{AE} \perp \overline{BC}$

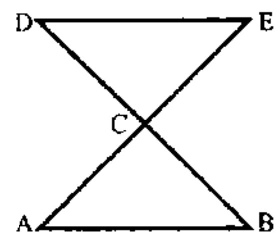
And $\overline{CD} \perp \overline{AB}$

To Prove: $m\overline{AE} \cong m\overline{CD}$

Proof: In $\Delta CAD \longleftrightarrow \Delta ACE$

Statements	Reasons
$m\angle CAD \cong m\angle ACE$	Angles opposite to congruent sides of triangle
$m\overline{AC} \cong m\overline{AC}$	Common
$m\angle CDA \cong m\angle AEC$	Both are of 90°
$\therefore \Delta CAD \cong \Delta ACE$	(A.A.S)
Thus $m\overline{AE} \cong m\overline{CD}$	Corresponding sides of congruent triangles

Q2: In the given figure $\overline{AC} \cong \overline{CE}$ and $\angle B \cong \angle D$ prove that $\overline{BC} \cong \overline{CD}$.



Given: $\overline{AC} \cong \overline{CE}$

And $\angle B \cong \angle D$

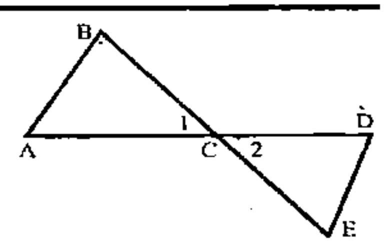
To Prove:

$\overline{BC} \cong \overline{CD}$

Proof:

Statements	Reasons
In $\Delta ABC \longleftrightarrow \Delta DEC$	
$\overline{AC} \cong \overline{CE}$	Given
$\angle B \cong \angle D$	Given
$\angle ACB \cong \angle DCE$	Vertical angles
$\therefore \Delta ABC \cong \Delta DEC$	(A.A.S),
So $\overline{BC} \cong \overline{CD}$	Corresponding sides of congruent triangles

Q3: Given: C is the midpoint of $\overline{BE}$. $\angle B \cong \angle C$, Prove: $\Delta ABC \cong \Delta DEC$.



Given: C is midpoint

And $\angle B \cong \angle E$

To Prove:

$\Delta ABC \cong \Delta DEC$

Proof:

Statements	Reasons
In $\Delta ABC \longleftrightarrow \Delta DEC$	
$m\overline{BC} \cong \overline{CE}$	Given $\therefore$ is mid point
$m\angle B \cong m\angle E$	Given
$m\angle 1 \cong m\angle 2$	Vertical angles
$\therefore \Delta ABC \cong \Delta DEC$	(A.A.S)

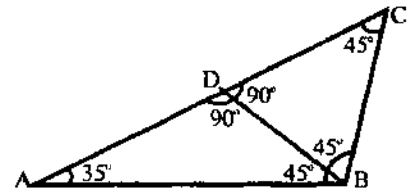
Q4: ABC is a triangle in which $m\angle A = 35°$ and $m\angle B = 100°$, $\overline{BD} \perp \overline{AC}$. Prove that ΔBDC is an isosceles triangle.

Given: ΔABC in which $m\angle A = 45^{\circ}$, $m\angle B = 100^{\circ}$

And $\overline{BD} \perp \overline{AC}$

To Prove:

$\overline{BD} \cong \overline{CD}$ or ΔBDC is isosceles triangle.



Proof:

Statements	Reasons
$\angle A = 35^{\circ}$, $\angle ABC = 100^{\circ}$	Given
$\overline{BD} \perp \overline{AC}$	Given
$m\angle ADB = \angle BDC = 90^{\circ}$	Both are supplementary angles
In ΔADB	
$m\angle ABD = 90^{\circ} - 35^{\circ} = 55^{\circ}$	
In ΔBDC	
$m\angle DBC = 100^{\circ} - 55^{\circ} = 45^{\circ}$	
So $m\angle BCD = 90^{\circ} - 45^{\circ} = 45^{\circ}$	
So in ΔBDC	Sides opposite to congruent angles
$\overline{BD} \cong \overline{CD}$	

Q5: Prove that the bisector of the vertex angle of an isosceles triangle is the perpendicular bisector of the base.

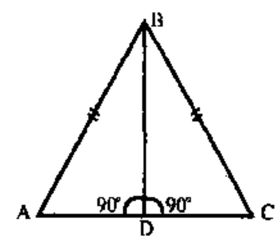
Given:

ΔABC in which $\overline{AC}$ is the base of ΔABC

And $\overline{BD} \perp \overline{AC}$

To Prove:

$\overline{AB} \cong \overline{BC}$



Proof:

Statements	Reasons
In $\Delta ABD \longleftrightarrow \Delta CBD$	
$\overline{BD} \cong \overline{BD}$	Common
$m\angle ADB \cong \angle CDB$	Each is right angle
$m\angle ABD \cong \angle CBD$	Given
$\therefore \Delta ABD \cong \Delta CBD$	(A.A.S)
Thus $\overline{AB} \cong \overline{BC}$	Corresponding sides of congruent ΔS

Q6: PQRS is a square. X, Y and Z are the midpoints of $\overline{PQ}, \overline{QR}$ and $\overline{RS}$ respectively. Prove that $\Delta PXY = \Delta SZY$.

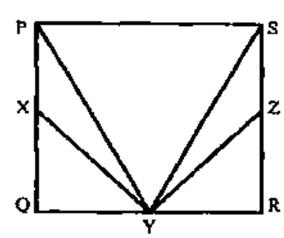
Given:

PQRS is a square in which X,

And Z are midpoints of $\overline{PQ}, \overline{QR}$

And $\overline{RS}$ respectively.

To Prove: $\Delta PXY \cong \Delta SZY$



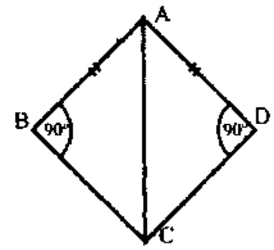
Proof:

Statements	Reasons
In $\Delta PXY \longleftrightarrow \Delta SZY$	
$\overline{PX} \cong \overline{SZ}$	Given X and Z are midpoints
$\overline{PY} \cong \overline{SY}$	Corresponding sides of congruent triangles isosceles Δ
$\overline{XY} \cong \overline{ZY}$	Corresponding sides of congruent triangles
$\therefore \Delta PXY \cong \Delta SZY$	S.S.S

Q7: Given: $\overline{AB} \perp \overline{BC}$, $\overline{AD} \perp \overline{DC}$, $\overline{AB} \cong \overline{AD}$. Prove that: $\Delta ABC \cong \Delta ADC$.

Given: $\overline{AB} \perp \overline{BC}$

$\overline{AD} \perp \overline{CD}$, $\overline{AB} \cong \overline{AD}$



To Prove:

$\Delta ABC \cong \Delta ADC$

Proof:

Statements	Reasons
In $\Delta ABC \longleftrightarrow \Delta ADC$	
$\overline{AB} \cong \overline{AD}$	Given
$m\angle ABC \cong m\angle ADC$	(Each is of 90^0) Given
$\overline{AC} \cong \overline{AC}$	Common
$\therefore \Delta ABC \cong \Delta ADC$	(S.A.S)

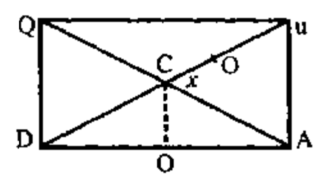
Q8: QUAD is a rectangle. Find x. $OC = x$, $DC = 3x - 8$

Solution:

If O is the midpoint of $\overline{AD}$, join O to C.

Then in ΔOCD, $(CD)^2 - (OC)^2 + (OD)^2$

Pythagoras theorem



$$(3x-8)^2 = (x)^2 + \left(\frac{x}{2}\right)^2$$

$$(3x-8)^2 = x^2$$

$$9x^2 + 64 - 48x = x^2$$

OR $9x^2 - x^2 - 48x + 64 = 0$

$8x^2 - 48x + 64 = 0$ (Divide by 8)

$x^2 - 6x + 8 = 0 \Rightarrow x^2 - 2x - 4x + 8 = 0$

$x(x-2) - 4(x-2) = 0 \Rightarrow (x-2)(x-4) = 0$

$x - 2 = 0$ OR $x - 4 = 0$

$\Rightarrow$ $x = 2$ OR $x = 4$

But $x \neq 2$

Hence $\boxed{x = 4}$ Ans.

Review Exercise 10

Q1: Select the correct answer.

i) Which of the following is not a sufficient condition for the congruency of two triangles?

(a) A.S.A ≅ A.S.A (b) H.S ≅ H.S

(c) S.A.A ≅ S.A.A ✓**(d) A.A.A ≅ A.A.A**

ii) The diagonal of...does not divide it into two congruent triangles.

(a) Rectangle (b) Square

(c) Parallelogram ✓**(d) Trapezium**

iii) In a given correspondence of two triangles, if $\Delta ABC \cong \Delta DEF$, then which of the following is not correct?

(a) $m\angle B = m\angle E$ (b) $m\overline{CA} \cong m\overline{FD}$

(c) $m\angle CBA \cong m\angle FED$ ✓**(d) $\angle ABC \cong \angle EFD$**

iv) In ΔABC, if $m\angle A \cong m\angle B$ then bisector of......divides the ΔABC into two congruent triangles.

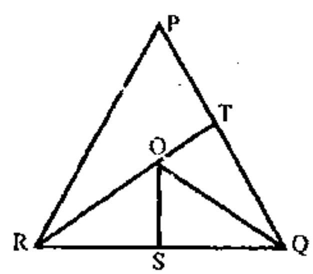
(a) $\angle A$ (b) $\angle B$ ✓**(c) $\angle C$** (d) Any one of its angles

v) Which of the following is a legitimate reason for declaring two triangles to be congruent?

(a) SSA ≅ SSA ✓**(b) SAS ≅ SAS**

(c) AAA ≅ AAA (d) All of these

Q2: In a ΔPQR, the bisectors of $\angle Q$ and $\angle R$ meet at O. Prove that O is equidistant from the sides of the triangle.



Solution: In ΔPQR, $\overline{OR}$ and $\overline{OQ}$ are the bisectors of $\angle R$ and $\angle Q$. Both bisectors meet at point O.

To Prove: $m\overline{OS} \cong m\overline{OT}$

Construction:

Draw $\overline{OS} \perp \overline{QR}$ and $\overline{OT} \cong \overline{PQ}$.

Proof:

Statements	Reasons
In $\Delta QTO \longleftrightarrow \Delta QSO$	
$m\angle QTO \cong \angle QSO$	Each is of 90°
$m\overline{OQ} \cong m\overline{OQ}$	Common
$m\angle OQS \cong m\angle OQT$	Given
$\therefore \Delta QTO \cong \Delta QSO$	(A.A.S)

So $m\overline{OS} \cong m\overline{OT}$	Corresponding sides of congruent triangles

Q3: In the given figure $\overline{AB} \cong \overline{CB}$ and $\angle A \cong \angle C$, prove that $\overline{AE} \cong \overline{CD}$.

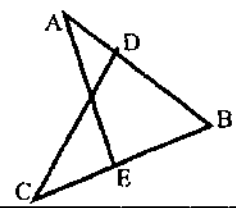
Solution:

$\overline{AB} \cong \overline{BC}$ and $\angle A \cong \angle C$

To Prove:

$\overline{AE} \cong \overline{CD}$

Proof:



Statements	Reasons
In $\Delta ABE \longleftrightarrow \Delta CBD$	
$m\angle A \cong m\angle C$	Given
$m\angle B \cong m\angle B$	Common
$m\overline{AB} \cong m\overline{BC}$	Given
$\therefore \Delta ABE \cong \Delta CBD$	(A.A.S)
So $\overline{AE} \cong \overline{CD}$	Corresponding sides of congruent triangles

Q4: ABC is a triangle in which $m\angle A = 35°$, $m\angle B = 100°$, $\overline{BD} \perp \overline{AC}$. Prove that ΔBDC is an isosceles triangle.

Given: ΔABC, in which $m\angle A = 35°$,

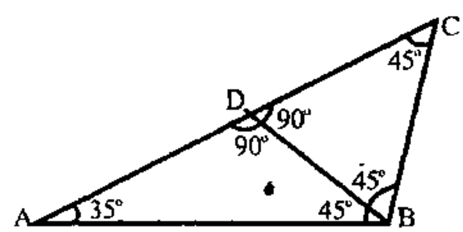
$m\angle B = 100°$

And $\overline{BD} \perp \overline{AC}$

To Prove: $\overline{BD} \cong \overline{CD}$

Or ΔBDC is an isosceles triangle

Proof:



Statements	Reasons
$\angle A = 35°, \angle ABC = 100°$	Given
$m\overline{BD} \perp m\overline{AC}$	Given
$m\angle ADB = \angle BDC = 90°$	Each is right angles
In ΔADB	
$m\angle ABD = 90° - 35° = 55°$	
$m\angle DBC = 100° - 55° = 45°$	
So in ΔBDC	
$m\overline{BD} = m\overline{CD}$	Sides opposite to congruent triangles

***

Additional MCQs of Unit 10:
Congruent Triangles

1. When the three sides and three angles are equal then the triangles are..........
(a) Parallel (b) Different (c) Congruent (d) none
✓ **Ans. (c) Congruent**

2. SAS postulate means two sides and..........
(a) One angle (b) Interior angle (c) Included angle (d) none
✓ **Ans. (c) Included angle**

3. $\Delta ABC \longleftrightarrow \Delta DEF$ shows..........between two triangles.
(a) Relation (b) Correspondence (c) Equality (d) none
✓ **Ans. (b) Correspondence**

4. If two angles and their included side are congruent then we write..........
(a) AAS (b) SAA (c) ASA (d) none
✓ **Ans. (c) ASA**

5. If two sides of a triangle are congruent then it is..........
(a) Isosceles (b) Scalene (c) Equilateral (d) none
✓ **Ans. (a) Isosceles**

6. A scalene triangle having three sides..........
(a) Unequal (b) Parallel (c) Perpendicular (d) Equal
✓ **Ans. (a) Unequal**

7. If hypotenuse and one side are congruent between two triangles then we write..........
(a) S.A.S (b) A.S.A (c) S.S.S (d) $H.L \cong H.L$
✓ **Ans. (d) $H.L \cong H.L$**

8. The diagonals of trapezium does not divide into two equal..........
(a) Squares (b) Triangles (c) Rectangles (d) none
✓ **Ans. (b) Triangles**

9. If two opposite sides are equal and parallel then it is called..........
(a) Quadrilateral (b) Rectangle (c) Trapezium (d) none
✓ **Ans. (c) Trapezium**

10. If $m\angle ABC = 90^o$ then such angle is called..........
(a) Acute (b) Abtuse (c) Right (d) none
✓ **Ans. (c) Right**

UNIT 11:
PARALLELOGRAMS & TRIANGLES

Definition:

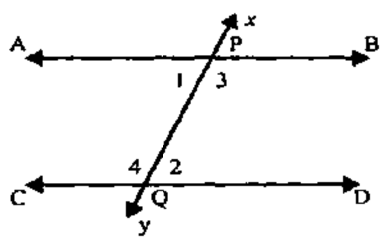
a) A quadrilateral is a closed figure having four sides and four angles. Line joining its opposite vertices is called its diagonal which divides the quadrilateral into two triangles.

b) Lines which are in the same plane and do not intersect each other, are called parallel lines.

c) If two lines lie in the same plane, they are said to be coplanar lines.

d) Lines drawn to cut two or more given lines is called transversal.

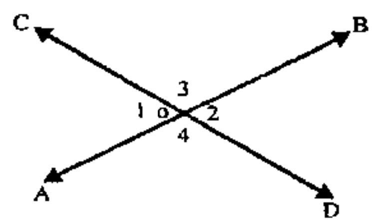
e) Line segment joining into midpoint of one side of a triangle to its opposite vertex is called the median of the triangle.



Alternate Angles:

When two coplanar lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are cut by a transversal $\overleftrightarrow{XY}$, two pairs of alternate angles are formed. If the coplanar lines are parallel, alternate angles are congruent.

From the figure, alternate angles $\angle 1 \cong \angle 2$ and $\angle 3 \cong \angle 4$



Vertical Angles:

When two lines intersect each other, the angle having a common vertex and having no common arm are called vertical angles. In the figure the two lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ intersect at O. $(\angle 1, \angle 2)$ and $(\angle 3, \angle 4)$ are two pairs of vertical angles. Also $\angle 1 \cong \angle 2$ and $\angle 3 \cong \angle 4$.

THEOREM 11.1

Statement: In a parallelogram:

i) The opposite sides are congruent

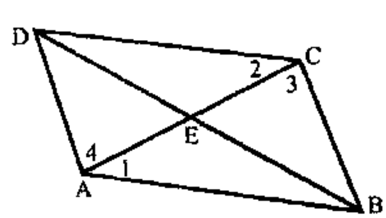
ii) The opposite angles are congruent

iii) The diagonals bisect each other

Given: ABCD is a parallelogram

i.e. $\overline{AB} \parallel \overline{DC}$ and $\overline{AD} \parallel \overline{BC}$

$\overline{AC}$ and $\overline{BD}$ are the diagonals of parallelogram



To Prove: $\overline{AB} \cong \overline{DC}$ and $\overline{AD} \cong \overline{BC}$

$\angle A \cong \angle C$ and $\angle B \cong \angle D$

$\overline{AE} \cong \overline{CE}$ and $\overline{DE} \cong \overline{BE}$

Proof:

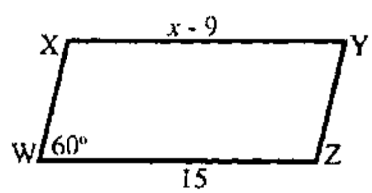
Statements	Reasons
$\because \overline{AB} \parallel \overline{DC}$ and $\overline{AC}$ intersects them	

$\therefore m\angle 1 \cong m\angle 2 \longrightarrow (1)$	Alternate angles
Similarly $\angle 3 \cong \angle 4 \longrightarrow (2)$	Alternate angles
In $\Delta ABC \longleftrightarrow \Delta CDA$	
$\overline{AC} \cong \overline{AC}$	Common
$m\angle 3 \cong m\angle 4$	Proved already
$m\angle 1 \cong m\angle 2$	Proved
$\therefore \Delta ABC \cong \Delta CDA$	(A.A.S)
Hence $\overline{AB} \cong \overline{CD}$	Corresponding sides of congruent triangles
And $\overline{BC} \cong DA$	Corresponding sides of congruent triangles
Also $\angle B \cong \angle D$	Corresponding angles of congruent triangles
$m\angle 1 + m\angle 4 \cong m\angle 2 + m\angle 3$	Adding (1) and (2)
Or $m\angle A \cong m\angle C$	$\angle 1 + \angle 4 = m\angle A$ and $\angle 2 + \angle 3 = m\angle C$
In $\Delta ABE \longleftrightarrow \Delta CDE$	
$\overline{AB} \cong \overline{CD}$	Proved
$m\angle 1 \cong m\angle 2$	Proved
$\angle AEB \cong \angle CED$	Vertical angles
$\therefore \Delta ABE \cong \Delta CDE$	(A.A.S $\cong$ A.A.S)
Hence $\overline{AE} \cong \overline{CE}$	
And $\overline{BE} \cong \overline{DE}$	Corresponding sides of congruent triangles

EXAMPLE 1

Quadrilateral *WXYZ* is a parallelogram. Find the value of *x* and *y*.

Solution:



$XY = WZ$ ($\because$ opposite sides of parallelogram are equal)

$x - 9 = 15$

$x = 24$

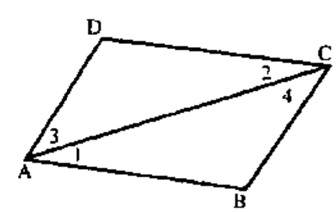
$y = 60$ ($\because$ opposite angles of a parallelogram are equal)

THEOREM 11.2

Statement: If two opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

Given: ABCD is a quadrilateral in which

$\overline{AB} \cong \overline{DC}$ and $\overline{AB} \cong \overline{DC}$



To Prove:

ABCD is a parallelogram.

Construction:

Join A to C.

Proof:

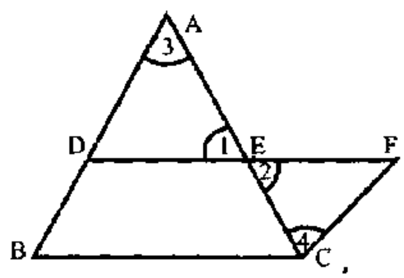
Statements	Reasons
In $\Delta ABC \longleftrightarrow \Delta CDA$	

$\overline{AB} \cong \overline{DC}$	Given
$\overline{AC} \cong \overline{AC}$	Common
$\angle 1 \cong \angle 2$	Alternate angles
$\therefore \Delta ABC \cong \Delta CDA$	(S.A.S $\cong$ S.A.S)
Hence $\angle 3 \cong \angle 4$	Corresponding angles of congruent triangles due to alternate angles
$\therefore \overline{AD} \parallel \overline{BC}$	
Hence ABCD is a parallelogram.	Opposite sides are parallel

THEOREM 11.3

Statement: The line segment, joining the midpoints of two sides of a triangle, is parallel to the third side and is equal to one half of its length.

Given: In $\Delta ABC, D$ and E are the midpoints of $\overline{AB}$ and $\overline{AC}$. $\overline{DE}$ joins them.



To Prove:

$\overline{DE} \parallel \overline{BC}$ and $m\overline{DE} = \frac{1}{2} m\overline{BC}$

Construction:

Takes F on $\overrightarrow{DE}$, such that $\overline{DE} \cong \overline{EF}$. Join F to C.

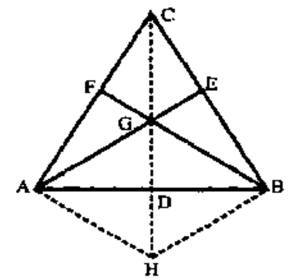
Proof:

Statements	Reasons
In $\Delta ADE \longleftrightarrow \Delta CFE$	
$\overline{DE} \cong \overline{EF}$	Construction
$\overline{AE} \cong \overline{CE}$	Given
$\angle 1 \cong \angle 2$	Vertical angles
$\therefore \Delta ADE \cong \Delta CFE$	(S.A.S $\cong$ (S.A.S)
$\therefore \overline{AD} \cong \overline{CF}$	Corresponding sides of congruent triangles
But $\overline{AD} \cong \overline{BD}$	Given
So $\overline{BD} \cong \overline{CF}$	Transitive property
Also $\overline{AB} \parallel \overline{CF}$	Alternate angles are congruent
Or $\overline{BD} \parallel \overline{CF}$	
i.e. BCFD is a parallelogram	Opposite sides are parallelogram
$\Rightarrow \overline{DE} \parallel \overline{BC}$	
And $m\overline{DF} = m\overline{BC}$	Opposite sides of parallelogram
But $m\overline{DE} = \frac{1}{2} m\overline{DF}$	Construction
$\therefore m\overline{DE} = \frac{1}{2} m\overline{BC}$	$(\because m\overline{DF} = m\overline{BC})$

Definition: A median of a triangle is a line segment from a vertex to the midpoint of the opposite side.

THEOREM 11.4

Statement: The medians of a triangle are concurrent and their point of concurrency is the point of trisection of each median.



Given: In $\Delta ABC, E$ and F are the midpoints of $\overline{BC}$ and $\overline{AC}$ respectively. $\overline{AE}$ and $\overline{BF}$ intersect each other at G. $\overrightarrow{CG}$ is drawn to meet $\overline{AB}$ at D.

To Prove: i) Medians are concurrent

ii) G is the point of trisection of each median

Construction: On $\overrightarrow{CG}$ take a point H such that $\overline{CG} \cong \overline{GH}$. Join H to A and B.

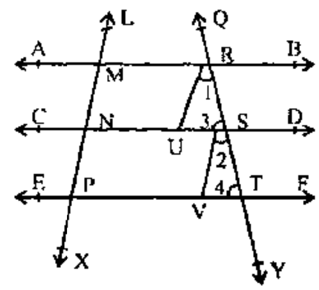
Proof:

Statements	Reasons
In ΔACH	$\overline{FG}$ joins midpoints of $\overline{CA}$ and $\overline{CH}$
$\overline{FG} \parallel \overline{AH}$	
i.e. $\overline{FB} \parallel \overline{AH}$	
Similarly in ΔBCH	G and E are midpoints of $\overline{CH}$ and $\overline{CB}$
$\overline{GE} \parallel \overline{HB}$	
Or $\overline{AE} \parallel \overline{HB}$	
$\therefore AHBG$ is a parallelogram	Opposite sides are parallel
i.e. $\overline{AD} \cong \overline{BD}$	Diagonals of a parallelogram bisect each other
Or D is the midpoint of $\overline{AB}$	
Or $\overline{CGD}$ is a median	
Hence all the three medians are concurrent	
Also $\overline{GD} \cong \overline{DH}$	Diagonals of a parallelogram bisect each other
Or $m\overline{GD} = \frac{1}{2} m\overline{GH} = \frac{1}{2} m\overline{GC}$	$\therefore \overline{CG} \cong \overline{GH}$ (construction)
Similarly it can be proved that	
$m\overline{GE} = \frac{1}{2} m\overline{AG}$	
And $m\overline{FG} = \frac{1}{2} m\overline{GB}$	
Hence G is the point of trisection of each median.	

Definition: The two lines which do not intersect each other and which are coplanar are called *parallel lines*. The two lines which intersect each other at right angle are called *perpendicular lines*.

THEOREM 11.5

Statement: If three or more parallel lines make congruent intercepts on a transversal, they also intercept congruent segments on any other line that cuts them.



Given: Lines $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD} \parallel \overleftrightarrow{EF}$.

The transversal $\overleftrightarrow{LX}$ intersects $\overleftrightarrow{AB}, \overleftrightarrow{CD}$ and $\overleftrightarrow{EF}$ at the points M, N and P such that $\overline{MN} \cong \overline{NP}$. Another transversal $\overleftrightarrow{QY}$ intersects them at points R, S, T respectively.

To Prove: $\overline{RS} \cong \overline{ST}$

Construction: Draw $\overline{RU} \parallel \overline{LX}$ which meets $\overleftrightarrow{CD}$ at U and from S draw $\overline{RV} \parallel \overline{LX}$ which meets $\overleftrightarrow{EF}$ at V. Label the angles as $\angle 1, \angle 2, \angle 3$ and $\angle 4$.

Proof:

Statements	Reasons
$\overline{RU} \parallel \overline{LX}$	Construction
$\overline{AB} \parallel \overline{CD}$	Given
$\therefore$ MNUR is a parallelogram	By definition
Thus $\overline{MN} \cong \overline{RU} \longrightarrow (1)$	Opposite sides of parallelogram
Similarly $\overline{NP} \cong \overline{SV} \longrightarrow (2)$	
But $\overline{MN} \cong \overline{NP} \longrightarrow (3)$	Given
$\therefore \overline{RU} \cong \overline{SV}$	From (1), (2) and (3)
So $RU \parallel \overline{SV}$	Both are congruent to $\overline{LX}$
Now in $\Delta RUS \longleftrightarrow \Delta SVT$	
$\overline{RU} \cong \overline{SV}$	Proved
$\angle 1 \cong \angle 2$	Corresponding angles
$\angle 3 \cong \angle 4$	Corresponding angles
$\therefore \Delta RUS \cong \Delta SVT$	(A.S.A $\cong$ A.S.A)
Hence $\overline{RS} \cong \overline{ST}$	Corresponding sides of the congruent triangle

EXERCISE 11.1

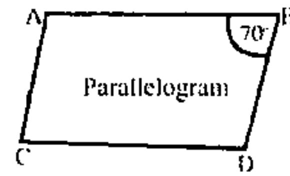
Q1: Measure of one of the angles of parallelograms is 70°. Find the measure of the remaining angles.

Given: ABCD is a parallelogram in which $m\angle B = 70°$.

To Prove: $m\angle C = 70°$

(Opposite angles of parallelogram are congruent)

$\angle A + \angle B + \angle C + \angle D = 360^\circ$ (Sum of all angles are 360°)



$\angle A + 70^\circ + 70^\circ + \angle D$

$\angle A + \angle D = 360^\circ - 70^\circ - 70^\circ$

$\angle A + \angle D = 220^\circ \Rightarrow \angle A = \angle D = \dfrac{220^\circ}{2} = 110^\circ$

Hence $m\angle C = 70^\circ$, $m\angle A = 110^\circ$ m and $m\angle D = 110^\circ$

Q2: Measure of one of the exterior angles of a parallelogram is 125°. Find the measure of all of its interior angles.

Solution: Given parallelogram $ABCD$ in which one exterior angle is $\angle BDX = 125^\circ$

i.e. $\angle BDX + \angle BDC = 180^\circ$ (Supplementary angles)

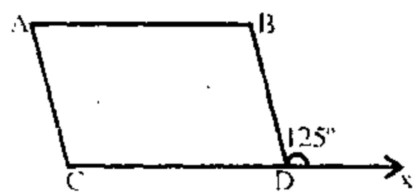
$125^\circ + \angle BDC = 180^\circ$

$\angle BDC = 180^\circ - 125^\circ = 55^\circ$ or $\angle D = 55^\circ$

So $\angle A = 55^\circ$ (Opposite angle of parallelogram)

$\angle A + \angle B + \angle C + \angle D = 360^\circ$ (Sum of all angles is 360°)

Putting the values of $\angle A = \angle D = 55^\circ$



$55^\circ + \angle B + \angle C + 55^\circ = 360^\circ$

$\angle B + \angle C + 110^\circ = 360^\circ$

$\Rightarrow \angle B + \angle C = 360^\circ - 110^\circ$

$\angle B + \angle C = 250^\circ$ or $\angle B = \dfrac{250^\circ}{2} = 125^\circ$

So $\angle C = 125^\circ$

Hence $\angle A = 55^\circ$, $\angle D = 55^\circ$, $\angle B = 125^\circ$ and $\angle C = 125^\circ$ Ans.

Q3: RSTU is a parallelogram. Find ST.

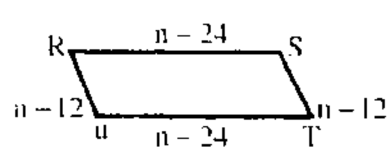
Solution: We know sum of all sides of parallelogram

$\Rightarrow n - 12 + n - 12 + n - 24 + n - 24 = 0$

$\Rightarrow 4n - 72 = 0$

$\Rightarrow 4n = 72$

$\Rightarrow n = \dfrac{72}{4} = 18$



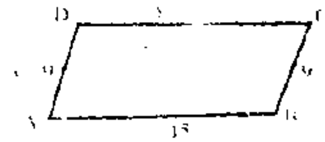
Now $ST = n - 12$

$\Rightarrow ST = 18 - 12 = 6$

Hence $\boxed{ST = 6}$ Ans.

Q4: Find the value of each variable in the parallelogram using the properties:

i)



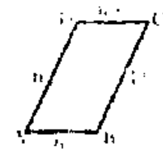
Solution:

Since ABCD is a parallelogram.

In parallelogram opposite sides are equal. Since $BC = AD$ and $AB = DC$

So $x = 9$, $y = 15$

ii)



Solution: Since $AB = DC$

$\Rightarrow m+1=6 \quad \Rightarrow m=6-1=5$

And $AD = BC$

$\Rightarrow \boxed{n=12}$ and $\boxed{m=5}$

iii)

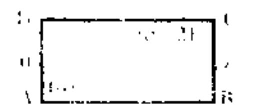
Solution: Since opposite angles of parallelogram are equal.

So $2P^\circ = 120$

$\Rightarrow P^\circ = \dfrac{120^\circ}{2} = 60^\circ$

$\boxed{P^\circ = 60^\circ}$ Ans.

iv)



Solution: Since $\overline{AD} = \overline{BC}$, so $Z-8=20$

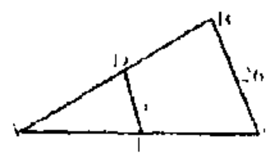
$\Rightarrow Z = 20+8 = 28$

And $\angle A = \angle C \Rightarrow (d-20)^\circ = 105^\circ$

$\Rightarrow d = 105^\circ + 20^\circ = 125$

Q5: $\overline{DE}$ is a mid segment of ΔABC. Find the value of x.

i)

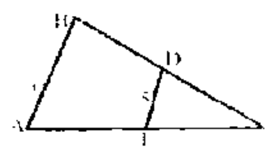


Solution: We know that,

$\overline{DE} = \dfrac{1}{2}\overline{BC}$ or $x = \dfrac{1}{2}(26)$

$\Rightarrow \boxed{x=13}$ Ans.

ii)

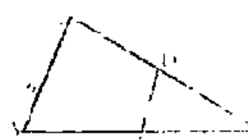


Solution: We know that,

$\overline{AB} = 2\overline{DE}$ or $x = 2(5)$

$\Rightarrow \boxed{x=10}$ Ans.

iii)

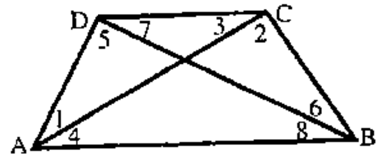


Solution: Since $\overline{DE} = \dfrac{1}{2}\overline{AB}$

$\Rightarrow x = \dfrac{1}{2}(6) \Rightarrow \boxed{x=3}$ Ans.

Q6: Prove that diagonals of a rhombus bisect its angles.

Solution: Let $\overline{AC}$ and $\overline{BD}$ be the two diagonals of the rhombus.



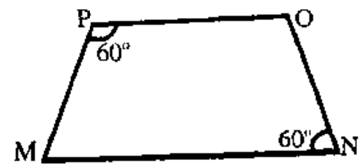
Since $\overline{AB} \parallel \overline{CD}$, so $\angle 3 \cong \angle 4$

And $\angle 1 \cong \angle 2$ because opposite angles of parallel sides are congruent.

Similarly $\angle 7 \cong \angle 8$ and $\angle 5 \cong \angle 6$

Hence the diagonals of a rhombus bisects the angles.

Q7: In rhombus *MNOP*, $m\angle N = 60^{\circ}$. What is $m\angle O$?



Solution: Since $m\angle N = 60^{\circ}$ and $\angle N \cong \angle P$

$\Rightarrow \angle P = 60^{\circ}$

But $\angle M + \angle N + \angle O + \angle P = 360^{\circ}$

(Sum of all angles are 360°)

$\angle M + 60^{\circ} + \angle O + 60^{\circ} = 360^{\circ}$

$\angle M + \angle O + 120^{\circ} = 360^{\circ}$

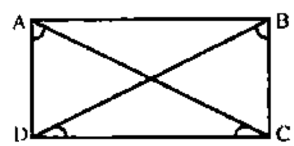
$\Rightarrow \angle M + \angle O = 360^{\circ} - 120^{\circ}$

$\angle M + \angle O = 240^{\circ}$

$\Rightarrow \angle M = \angle O = \frac{240^{\circ}}{2} = \boxed{120^{\circ}}$

Hence $\boxed{m\angle O = 120^{\circ}}$ Ans.

Q8: Prove that diagonals of a rectangle are congruent.



Given: ABCD is a rectangle in which $\overline{AB} \cong \overline{DC}$

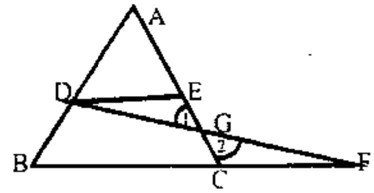
And $\overline{AD} \cong \overline{BC}$.

To prove: $\overline{AC} \cong \overline{BD}$

Proof:

Statement	Reasons
In $\Delta ADC \longleftrightarrow \Delta BCD$	
$\overline{CD} \cong \overline{CD}$	Common
$m\angle D \cong m\angle C$	Each is 90°
$\overline{AD} \cong \overline{BC}$	Given
$\therefore \Delta ADC \cong \Delta BCD$	(A.A.S)
Hence $\overline{AC} \cong \overline{BD}$	Corresponding sides

Q9: In a $\Delta ABC, D$ and E are two points on $\overline{AB}$ and $\overline{AC}$, such that $m\overline{AD} = \frac{1}{4} m\overline{AB}$ and $m\overline{AE} = \frac{1}{4} m\overline{AC}$. Prove that $m\overline{DE} = \frac{1}{4} m\overline{BC}$.



Given: ΔABC in which $\overline{AD} = \frac{1}{4}\overline{AB}$, $\overline{AE} = \frac{1}{4}\overline{AC}$

To Prove: $\overline{DE} = \frac{1}{4}\overline{BC}$

Construction: Join F with G and D such that

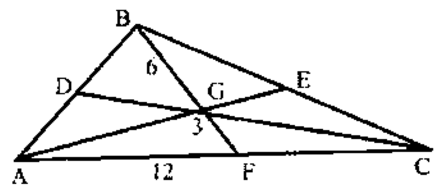
$m\overline{DG} \cong m\overline{GF}$ and $\overline{GC} \cong m\overline{GE}$. Produce $\overline{CF}$ such that $\overline{CF} = \frac{1}{4}\overline{BC}$.

Proof:

Statement	Reasons
In $\Delta DEG \longleftrightarrow \Delta CGF$	
$\overline{DG} \cong \overline{GF}$	Construction
$\overline{GE} \cong \overline{GC}$	Construction
$m\angle 1 \cong m\angle 2$	Vertical angles
$\therefore \Delta DEG \cong \Delta CGF$	S.A.S $\cong$ S.A.S
So $\overline{DE} \cong \overline{CF}$	Corresponding sides
But $CF = \frac{1}{4}\overline{BC}$	Construction
$\therefore \overline{DE} \cong \frac{1}{4}\overline{BC}$	Transitive property

Q10: G is the centroid of $\Delta ABC, BG = 6, AF = 12$ and $AE = 15$. Find the length of the segment.

i) $\overline{FC}$ **ii)** $\overline{BF}$
iii) $\overline{AG}$ **iv)** $\overline{GE}$



Solution:

i) $\overline{FC}$

Since $\overline{BF}$ is the median of $\overline{AC}$ so $\overline{AF} = \overline{FC}$

Hence $\overline{FC} = 12$

ii) $\overline{BF}$

Since $BG : GF = 2:1$ or $BG : GF = 6:3$

Hence $\overline{BF} = \overline{BG} + GF = 6 + 3 = 9$

iii) $\overline{AG}$

Since $\overline{AE} = \overline{AG} + GE = 10 + 2 = 12$

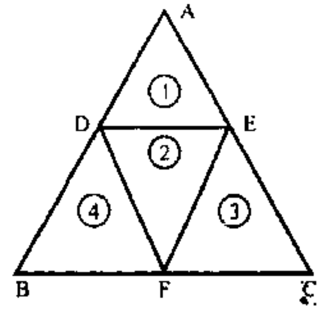
Hence $\overline{AG} = 10$

iv) $\overline{GE}$

Since $\overline{AG} : \overline{GE} = 10 : 2 \Rightarrow \overline{GE} = 2$ Ans.

Q11: Prove that the four triangles formed by joining the midpoints of the three sides of a triangle are congruent to one another.

Given: In ΔABC points D, E, F are midpoints of $\overline{AB}, \overline{AC}, \overline{BC}$. Four triangles are formed by joining these points.



Proof: $\overline{DE} \parallel \overline{BC}$ or $\overline{DE} \parallel \overline{BF}$

Or $m\overline{DE} = \frac{1}{2} m\overline{BL}$ or $\overline{DE} \cong m\overline{BF}$

So BFED is a parallelogram.

So $\Delta BFD \cong \Delta DEF$ $\because \overline{DE} \cong \overline{BF}$ and $\overline{BD} \cong \overline{EF}$
Similarly $\Delta ADE \cong \Delta DEF$ and $\Delta CEF \cong \Delta DEF$
Hence all four triangles are congruent.

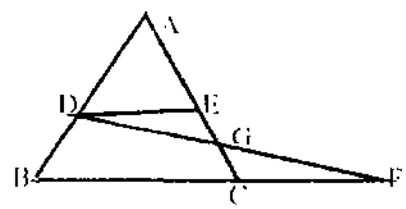
Q12: In the given $\Delta ABC, D$ and E are the midpoints of $\overline{AB}$ and $\overline{AC}$, and $m\overline{CF} = \frac{1}{2} m\overline{BC}$.

Solution: Given: In ΔABC

$\overline{AD} = \overline{BD}$

And $\overline{AE} \cong \overline{EC}$

To Prove: $m\overline{CG} = \frac{1}{3} m\overline{AG}$



Proof:

Statement	Reasons
In ΔABC	
$\overline{DE} \parallel \overline{BC}$	$\overline{DE}$ joins the midpoint of ΔABC
$\angle EDG \cong \angle CFG$	Alternate angles of parallel sides
$\therefore$ $m\overline{DE} = \frac{1}{2} m\overline{BC} \longrightarrow (1)$	
$\overline{DE} = \overline{BC}$	Given
$\therefore$ $m\overline{CF} = \frac{1}{2} m\overline{BC} \longrightarrow (2)$	
$\overline{DE} \parallel \overline{BE}$	From equation (1) and equation (2)
$\therefore$ $\overline{DE} \cong \overline{CF}$	
In $\Delta GED \longleftrightarrow CFG$	
$\angle EDG \cong \angle CFG$	Proved above
$\angle DGE \cong \angle FGE$	Vertical angles
$\overline{DE} \cong \overline{CF}$	Proved
$\therefore$ $\Delta GED \cong \Delta CFG$	
Hence $\overline{EG} \cong \overline{GC}$	Corresponding sides
$\overline{AE} \cong \overline{CE}$	Corresponding sides
$\overline{CG} + \overline{CG} = \overline{CE}$	
$2\overline{CG} = \overline{CE} \longrightarrow (1)$	
Now $\overline{AG} \cong \overline{CE} + \overline{EG}$	From $\overline{CE} \cong 2\overline{CG} + \overline{EG} = \overline{GC}$
$\overline{AG} \cong \overline{CG} + \overline{EG}$	
$\overline{AG} \cong 2\overline{CG} + \overline{GC}$	
$\overline{AG} \cong 3\overline{CG}$	
$\overline{CG} = \frac{1}{3} \overline{AG}$	

REVIEW EXERCISE 11

Q1: Select the correct answers:

i) Which quadrilateral must have diagonals that are congruent and perpendicular?
(a) Rhombus ✓**(b) Square**
(c) Trapezoid (d) Parallelogram

ii) How many equilateral tri angles can be made by joining the midpoints of the sides of an equilateral triangle?
(a) 2 (b) 3 ✓**(c) 4** (d) Cannot be determined

iii) Which statement must be true?
✓**(a) If a parallelogram is not a rectangle, then it is not a square**
(b) If a parallelogram is not a square, then it is not a rectangle
(c) All rectangles are squares
(d) If a parallelogram is a rectangle, then it is a square

iv) Which quadrilateral's diagonals are perpendicular to each other and bisect the figure's opposite angles?
(a) Trapezoid (b) Rectangle
✓**(c) Rhombus** (d) Parallelogram

v) If opposite angles of a quadrilateral are equal in measure and none of them is a right angle, then the quadrilateral is a:
(a) Square ✓**(b) Parallelogram**
(c) Trapezoid (d) Rectangle

vi) Medians of a triangle are divided by the point of concurrency in the ratio:
✓**(a) 2 : 1** (b) 2 : 3 (c) 1 : 3 (d) None of these

vii) Centroid is the point of concurrency is:
✓**(a) Medians of a triangle** (b) Angle bisectors of a triangle
(c) Altitudes of a triangle (d) Perpendicular bisectors of a triangle

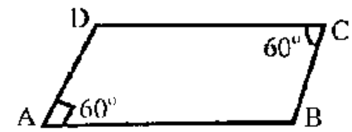
viii) If sum of the measures of $\angle A$ and $\angle C$ of a parallelogram ABCD is 120°, then $m\angle B = \ldots\ldots$
(a) 25° (b) 50° (c) 65° ✓**(d) None of these**

ix) Diagonals of a square are....to each other.
✓**(a) Perpendicular** (b) Not congruent
(c) Congruent (d) Parallel

x) Sum of the measures of interior angles of a quadrilateral is:
(a) 2 right angles ✓**(b) 4 right angles**
(c) 3 right angles (d) None of these

Q2: Measure of one of the angles of parallelogram is 60^0. Find the measure of the remaining angles.

Solution:



Since $\angle A = 60^o \quad \Rightarrow \angle A \cong \angle C = 60^o$

$\Rightarrow \angle A + \angle B + \angle C + \angle D = 360^o$

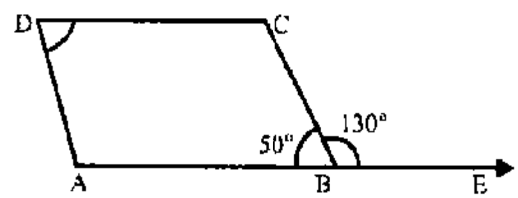
$\Rightarrow 60^o + \angle B + 60^o + \angle D = 360^o \qquad \therefore \angle B = \angle D = 120^o$

$\Rightarrow \angle B + \angle D + 120^o = 360^o$

$\Rightarrow \angle B = \dfrac{240^o}{2} = 120^o$

Q3: Measure of one of the exterior angles of a parallelogram is 130^0. Find the measure of all of its interior angles.

Solution:



Since $\angle CBE = 130^o$

So $\angle ABC + \angle CBE = 180^o$

$\Rightarrow \angle ABC + 130^o = 180^o$

$\Rightarrow \angle ABC = 180^o - 130^o = 50^o$

$\therefore \angle D \cong \angle B = 50^o$

Now $\angle A + \angle B + \angle C + \angle D = 360^o$

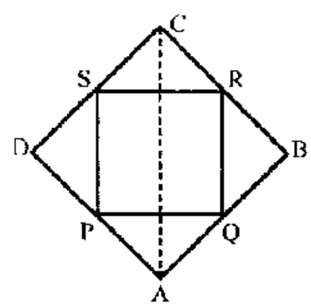
$\Rightarrow \angle A + 50^o + \angle C + 50^o = 360^o$

$\Rightarrow \angle A + \angle C = 360^o - 100^o = 260^o$

$\Rightarrow \angle A = \dfrac{260^o}{2} = 130^o$

$\Rightarrow \angle C = 130^o$

Q4: Prove that the line segments joining the midpoints of the sides of a quadrilateral taken in order, form a parallelogram.



Given: ABCD is a quadrilateral points P, Q, R, S are the mid-points of $\overline{AD}, \overline{AB}, \overline{BC}, \overline{CD}$.

To Prove: PQRS is a parallelogram.

Construction: Draw diagonal $\overline{AC}$.

Proof: In ΔADC, P and S are midpoints of $\overline{AD}$ and $\overline{CD}$.

So $\overline{PS} \parallel \overline{AC} \longrightarrow (1)$

$\Rightarrow m\overline{PS} = \dfrac{1}{2} m\overline{AC} \longrightarrow (2)$

In ΔABC, Q and R are midpoints of $\overline{AB}$ and $\overline{BC}$

So $\overline{QR} \parallel \overline{AC} \longrightarrow (3)$

And $m\overline{QR} = \dfrac{1}{2} m\overline{AC} \longrightarrow (4)$

$\therefore \overline{PS} \parallel \overline{QR}$ from (1) and (3) and $\overline{PS} \parallel \overline{QR}$

From (2) and (4) similarly $\overline{PQ} \parallel \overline{RS}$

Hence PQRS is a parallelogram.

Q5: In rhombus $MNOP$, $m\angle N = 70^\circ$, what is $m\angle O$?

Solution: Given $m\angle N = 70^\circ$

As $m\angle N = m\angle P$, so $m\angle P = 70^\circ$

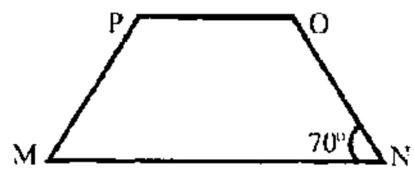
Now $\angle M + \angle N + \angle O + \angle P = 360^\circ$ (Sum of all angles are 360°)

$\angle M + 70^\circ + \angle O + 70^\circ = 360^\circ$

$\angle M + \angle O + 140^\circ = 360^\circ$

$\angle M + \angle O = 360^\circ - 140^\circ = 220^\circ$

OR $\angle O = \dfrac{220^\circ}{2} = 110^\circ$



$\therefore$ $\boxed{\angle M = \angle O = 110^\circ}$ Ans.

Q6: Measure of one of the angles of parallelogram is 50°. Find the measure of the remaining angles.

Solution:

Given ABCD is a parallelogram

Such that $m\angle A = 50^\circ$

Required:

$\angle B, \angle C, \angle D$

Proof:

	Statement	Reasons
	$\overline{AD} \parallel \overline{BC}$	Opposite sides
	$m\angle A + m\angle B = 180^\circ$	Supplementary angles
	$m\angle A = 50^\circ$	Given
$\Rightarrow$	$m\angle B = 180^\circ - 50^\circ = 130^\circ$	
	$m\angle A = m\angle C = 50^\circ$	Opposite angles of parallelogram
	$m\angle B = m\angle O = 130^\circ$	Opposite angles of parallelogram
Hence	$m\angle B = 130^\circ$,	
	$m\angle O = 130^\circ$,	
	$m\angle C = 50^\circ$	

***

Additional MCQs of Unit 11:
Parallelograms and Triangles

1. A polygone with four sides is called..........
(a) Quadrilateral (b) Rectangle (c) Square (d) none
✓ **Ans. (a) Quadrilateral**

2. In geometry, a figure that lies in a plane is called..........figure.
(a) Space (b) Cartesian (c) Plane (d) none
✓ **Ans. (c) Plane**

3. When all four sides are congruent then the figure is..........
(a) Rectangle (b) Rhombus (c) Square (d) none
✓ **Ans. (c) Square**

4. A quadrilateral in which all sides and all angles are congruent is called..........
(a) Rhombus (b) Square (c) Rectangle (d) none
✓ **Ans. (b) Square**

5. If two opposite sides of a quadrilateral are congruent then it is..........
(a) Square (b) Rectangle (c) Parallelogram (d) none
✓ **Ans. (c) Parallelogram**

6. The diagonal divides which quadrilateral into two congruent triangles..........
(a) Square (b) Rectangle (c) Parallelogram (d) All
✓ **Ans. (d) All**

7. The line segment joining the midpoints of two sides of a triangle is..........to third side.
(a) Parallel (b) Perpendicular (c) Intersect (d) none
✓ **Ans. (a) Parallel**

8. A line segment from a vertex to the midpoint of the opposite side of a triangle is..........
(a) Bisector (b) Altitude (c) Median (d) none
✓ **Ans. (c) Median**

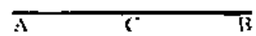
9. The medians of a triangle are..........
(a) Different (b) Concurrent (c) Proportional (d) none
✓ **Ans. (b) Concurrent**

10. The two lines which do not intersect and are Coplanar are ....lines.
(a) Parallel (b) Perpendicular (c) Horizontal (d) none
✓ **Ans. (a) Parallel**

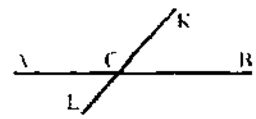
UNIT 12:
LINE BISECTORS & ANGLE BISECTORS

Definition:

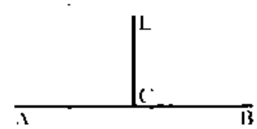
a) If C is a point on $\overline{AB}$ such that $m\overline{AC} = m\overline{BC}$, then C is said to be the midpoints of $\overline{AB}$.



b) Any line through C (midpoint of $\overline{AB}$) is called the bisector of $\overline{AB}$ i.e. $\overleftrightarrow{KL}$ is the bisector of $\overline{AB}$.

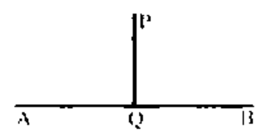


c) If a bisector of a line segment is perpendicular to the line segment as well, then it is said to be 'right-bisector'. In the figure $\overleftrightarrow{LC}$ is the right bisector of $\overline{AB}$ because it passes through its midpoint C and is also perpendicular to $\overline{AB}$.



d) If a point D exists in the interior of an angle ABC, such that $m\angle ABD = m\angle DBC$ then $\overrightarrow{BD}$ is called the bisector of angle ABC.

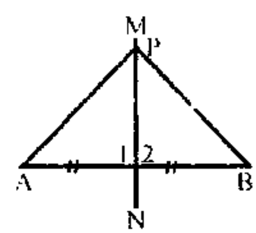
e) Distance of a point from a line is the length of perpendicular drawn from the point to the line. Distance of P from $\overline{AB}$ is $m\overline{PQ}$, where $\overline{PQ} \perp \overline{AB}$.



THEOREM 12.1

Any point on the right bisector of a line segment is equidistant from end points of the segment.

Given: A line $\overleftrightarrow{MN}$ intersects the line segment $\overline{AB}$ at the point R such that $\overleftrightarrow{MN}$ is the right bisector of $\overline{AB}$ which cuts $\overline{AB}$ at R. Let P be a point on $\overleftrightarrow{MN}$.



Construction:

Join P to points A and B.

To Prove: P is equidistant from A and B i.e. $m\overline{PA} = m\overline{PB}$

Proof:

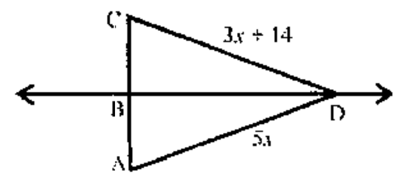
Statements	Reasons
In $\Delta PRA \longleftrightarrow \Delta PRB$	
$\overline{PR} \cong \overline{PR}$	Common
$\overline{AR} = \overline{BR}$	Given
$\angle 1 \cong \angle 2$	Right angles
$\therefore$ $\Delta PRA \cong \Delta PRB$	(S.A.S $\cong$ S.A.S)
Hence $\overline{PA} \cong \overline{PB}$	Corresponding sides of congruent triangles
This means that P is equidistant from A and B.	

EXAMPLE

$\overleftrightarrow{BD}$ is the perpendicular bisector of $\overline{AC}$. Find AD.

Solution:

$AD = CD$	perpendicular bisector theorem
$5x = 3x + 14$	substitute.
$x = 7$	solve for x
$AD = 5x$	
$= 5(7)$	
$= 35$	



THEOREM 12.2

Any point equidistant from the end points of a line segment is on the right bisector of it.

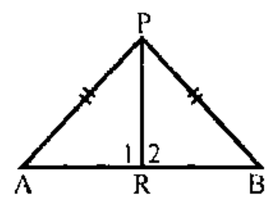
Given: P is equidistant from the end points of $\overline{AB}$ i.e. $\overline{PA} \cong \overline{PB}$

To Prove:

P lies on the right bisector of $\overline{AB}$.

Construction:

Draw a perpendicular on $\overline{AB}$, meeting $\overline{AB}$ at R.



Proof:

Statements	Reasons
In $\Delta PRA \longleftrightarrow \Delta PRB$	
$\overline{PA} \cong \overline{PB}$	Given
$\overline{PR} \cong \overline{PR}$	Common
$m\angle 1 \cong m\angle 2 = 90^\circ$	Construction
$\therefore$ $\Delta PRA \cong \Delta PRB$	(S.A.S $\cong$ S.A.S)
Hence $\overline{PR} \cong \overline{PR}$	Corresponding sides of congruent triangle
$\therefore$ $\overline{PR}$ is the right bisector of $\overline{AB}$ or P lies on the right bisector of $\overline{PR}$	

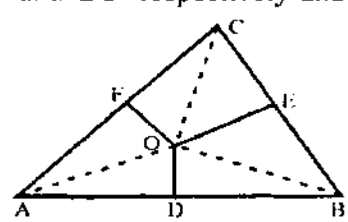
THEOREM 12.3

Statement: The right bisectors of the sides of a triangle are concurrent.

Given: In ΔABC, $\overline{DO}$ and $\overline{EO}$ are the right bisectors of $\overline{AB}$ and $\overline{BC}$ respectively and intersects each other at O. Join O to F which is midpoint of $\overline{AC}$.

To Prove: The right bisectors of the sides $\overline{AB}$, $\overline{BC}$, $\overline{AC}$ of a triangle are concurrent at O.

Construction: Draw the right bisectors of $\overline{AB}$ and $\overline{BC}$ which intersect at O. Join O to A, B, C.



Proof:

Statements	Reasons
In ΔAOB	

O lies on the right bisector of $\overline{AB}$	Given
$\therefore \quad \overline{AO} \cong \overline{BO} \longrightarrow (1)$	O is on right bisector of $\overline{AB}$
Similarly in ΔBOC	
$\overline{BO} \cong \overline{CO} \longrightarrow (2)$	
Hence $\overline{AO} \cong \overline{CO}$	From (1) and (2)
Or O is on the right bisector of $\overline{AC}$ Hence the right bisectors of three sides of a triangle are concurrent at point O.	O is equidistant from A and C

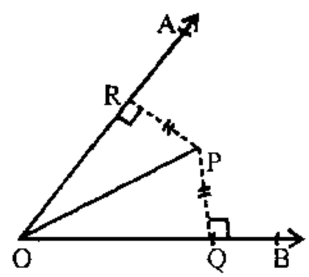
THEOREM 12.4

Statement: Any point on the bisector of an angle is equidistant from its arms.

Given: $\overrightarrow{OM}$ is the bisector of $\angle AOB$.
P is any point on $\overrightarrow{OM}$.

To Prove:
P is equidistant from the arms of $\angle AOB$ i.e. $\overline{PR} \cong \overline{PQ}$

Construction:
Draw $\overline{PR} \perp \overline{OA}$ and $\overline{PQ} \perp \overline{OB}$.



Proof:

Statement	Reasons
In $\quad \Delta PRO \longleftrightarrow \Delta POQ$	
$\angle POR \cong \angle POQ$	Given
$\angle PQO \cong \angle PRO$	Right angles
$\overline{PO} \cong \overline{PO}$	Common
$\therefore \quad \Delta PRO \cong \Delta POQ$	(A.A.S $\cong$ A.A.S)
Hence $\overline{PR} \cong \overline{PQ}$	Corresponding sides of congruent triangles
i.e. P is equidistant from $\overrightarrow{BA}$ and $\overrightarrow{BC}$.	

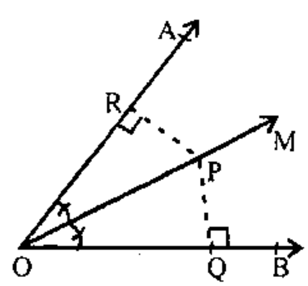
THEOREM 12.5

Statement: Any point inside an angle, equidistant from its arms, is on its bisector.

Given: P is a point lies inside $\angle AOB$,
Such that $\overline{PR} \cong \overline{PQ}$ where $\overline{PR} \perp \overline{OA}$ and $\overline{PQ} \perp \overline{OB}$

To Prove: $\overline{OP}$ lies on the bisector of $\angle AOB$

Construction:
Joint P to O. Draw perpendicular $\overline{PR}$ and $\overline{PQ}$ on $\overline{OA}$ and $\overline{OB}$.



Proof:

Statements	Reasons
In $\Delta OPR \longleftrightarrow \Delta OPQ$	
$\overline{PO} \cong \overline{PO}$	Common
$\overline{PR} \cong \overline{PQ}$	Given
$\angle PRO \cong \angle PQO$	Right angles
$\therefore$ $\Delta OPR \cong \Delta OPQ$	(S.A.S $\cong$ S.A.S)
Hence $\angle POR \cong \angle POQ$	Corresponding angles of the congruent triangle
Or $\overline{OP}$ is the bisector of $\angle AOB$	

EXAMPLE:

For what values of x does P lies on the bisector of $\angle A$?

Solution:

From the converse of the angle bisector theorem P lies on the bisector of $\angle A$ if P is equidistant from the sides of $\angle A$, so when $BP = CP$.

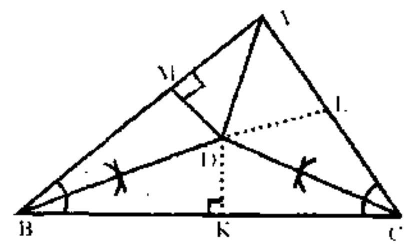
$BP = CP$	set segment lengths equal
$x + 3 = 2x - 1$	substitute expressions for segments lengths
$4 = x$	solve for x

Thus the point P lies on the bisector of $\angle A$ when $x = 4$.

An angle bisector of a triangle is the bisector of the interior angle of the triangle.

THEOREM 12.6

Statement: The bisectors of the angles of a triangle are concurrent.



Given: $\overline{BD}$ and $\overline{CD}$ are the bisectors of $\angle B$ And $\angle C$ of ΔABC, which intersects each other at D. Join D with A.

To Prove: $\overline{DA}$ is bisector of $\angle A$.

Construction:

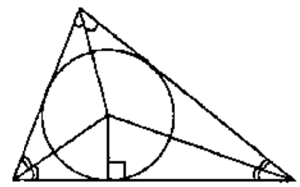
From D, draw $\overline{DK} \perp \overline{BC}$, $\overline{DL} \perp \overline{CA}$ and $\overline{DM} \perp \overline{AB}$.

Proof:

Statements	Reasons
As D lies on bisector of $\angle B$	Given
$\therefore$ $\overline{DK} \cong \overline{DM} \longrightarrow (1)$	Distance of D from the arm of $\angle B$
Similarly D lies on the bisector of $\angle C$	
$\therefore$ $\overline{DK} \cong \overline{DL} \longrightarrow (2)$	Distance of D from the arms of $\angle C$
Hence $\overline{DL} \cong \overline{DM}$	From (1) and (2)
i.e. D lies on the bisector of $\angle A$	D is equidistant from L and M
Or $\overline{AD}$ is the bisector of $\angle A$	
Or the bisectors of the angles of the ΔABC are concurrent.	

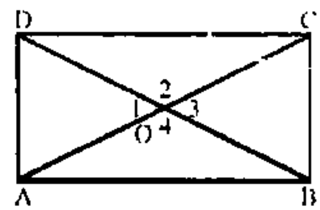
Definition:

The point of concurrency of the three angle bisectors of a triangle is called the "Incentre" of the triangle. The "Incentre" always lies inside of the triangle. The "Incentre" is equidistant from the three sides of the triangle.



EXERCISE 12.1

Q1: If the diagonals of a quadrilateral are the right bisectors of each other, then prove that all the sides of the quadrilateral are congruent.



Given: ABCD is a quadrilateral. $\overline{AC}$

And $\overline{BD}$ are the two diagonals meet at O

Such that $\overline{OA} = \overline{OC} = \overline{OB} = \overline{OD}$

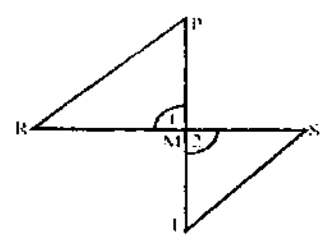
$m\angle 1 \cong m\angle 2 = m\angle 3 \cong m\angle 4$

To Prove: $\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{AD}$

Proof:

Statements	Reasons
In $\Delta BOC \longleftrightarrow \Delta DOC$	
$\overline{OC} \cong \overline{OC}$	Common
$\overline{OB} \cong \overline{OD}$	Given
$m\angle 2 \cong m\angle 3$	Given
$\therefore$ $\Delta BOC \cong \Delta DOC$	(S.A.S $\cong$ S.A.S)
So $\overline{BC} \cong \overline{CD}$	Corresponding sides of congruent triangles
Similarly we can prove that $\overline{CD} \cong \overline{DA}$ and $\overline{DA} \cong \overline{AB}$	

Q2: If $\overline{PR}$ and $\overline{TS}$ are $\perp$ $\overline{RS}, \overline{PM} \cong \overline{MT}$ and $m\angle PMT = 90^\circ$. Prove that $\Delta PRM \cong \Delta MTS$.



Given:

$\overline{PM} \cong \overline{MT}$

$m\angle 1 \cong m\angle 2 = 90^\circ$

$\overline{RM} \cong \overline{MS}$

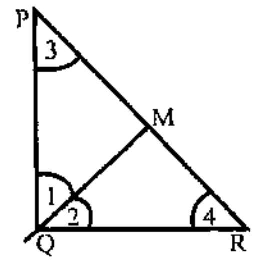
To Prove:

$\Delta PRM \cong \Delta MTS$

Proof:

Statements	Reasons
In $\Delta PRM \longleftrightarrow \Delta MTS$	
$\overline{PM} \cong \overline{MT}$	Given
$m\angle 1 \cong m\angle 2$	Each is 90° (Given)
$\overline{RM} \cong \overline{MS}$	Given
$\therefore$ $\Delta PRM \cong \Delta MTS$	S.A.S $\cong$ S.A.S

Q3: If $\angle 3 \cong \angle 4$ and $\overline{QM}$ bisectors $\angle PQR$, prove that M is the midpoint of $\overline{PR}$.



Given:

$\angle 3 \cong \angle 4$, QM bisectors $\angle PQR$

Such that $\angle 1 \cong \angle 2$

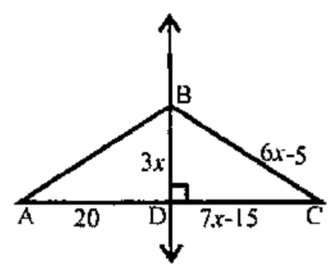
To Prove:

$\overline{PM} \cong \overline{MR}$ or M is the midpoint of $\overline{PR}$.

Proof:

Statements	Reasons
In $\Delta PMQ \longleftrightarrow \Delta MQR$	
$m\angle 1 \cong m\angle 2$	Given
$m\angle 3 \cong m\angle 4$	Given
$\overline{MQ} \cong \overline{MQ}$	Common
$\therefore$ $\Delta PMQ \cong \Delta MQR$	(A.A.S $\cong$ A.A.S)
Hence $\overline{PM} \cong \overline{MR}$	Corresponding sides

Q4: Find the length of AB.



Solution:

From the figure, $\Delta ABD \cong \Delta BDC$ and both are right triangles.

So $\overline{AB} \cong \overline{BC}$ = hypotenuse

Now $(AB)^2 = (BC)^2 \Rightarrow (5x)^2 = (4x+3)^2$

$25x^2 = 16x^2 + 24x + 9$

OR $25x^2 - 16x^2 - 24x - 9 = 0 \qquad \Rightarrow 9x^2 - 24x - 9 = 0$

$3(3x^2 - 8x - 3) = 0 \qquad \Rightarrow 3x^2 - 8x - 3 = 0$

$\Rightarrow 3x^2 - 9x + x - 3 = 0 \qquad \Rightarrow 3x(x-3) + 1(x-3) = 0$

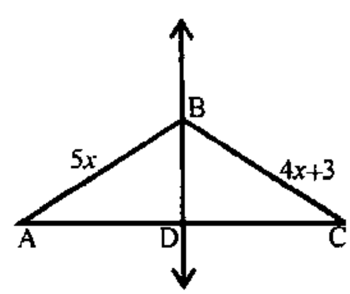
$(x-3)(3x+1) = 0$

$\therefore x - 3 = 0 \Rightarrow x = 3$

Now length of $\overline{AB} = 5x = 5(3) = 15$ Ans.

Q5: In the diagram, $\overleftrightarrow{BD}$ is the perpendicular bisector of $\overline{AC}$.

i) **What segment lengths are equal?**
ii) **What is the value of x?**
iii) **Find AB.**



Solution:

In $\Delta ABD \longleftrightarrow \Delta BDC$

$\overline{AD} = \overline{DC}$

$\because \overline{BD}$ is perpendicular bisector of $\overline{AC}$

So $\overline{AB} = \overline{BC}$ (Each is hypotenuse of same triangles)

Now $\overline{AD} = \overline{DC} \Rightarrow 7x - 15 = 20$

$\Rightarrow 7x = 20 + 15 = 35$

$\Rightarrow x = \frac{35}{7} = 5$

i) Here $\overline{AB} = \overline{BC}$ are equal segment

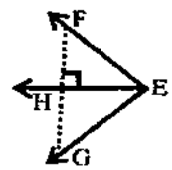
ii) $x = 5$

iii) $\overline{AB} = 6x - 5 = 6(5) - 5 = 30 - 5 = 25$ Ans.

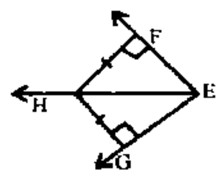
Q6: Can we conclude that $\overline{EH}$ bisects $\angle FEG$?

Solution:

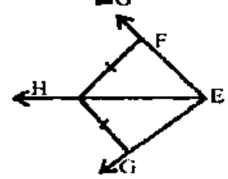
i) No, we cannot conclude that $\overline{EH}$ is the bisector of $\angle FEG$ because $\angle FHE = 90^o$ but $\angle GHE \neq 90^o$



ii) Yes, here we can conclude that $\overline{EH}$ bisects $\angle FEG$ because $\overline{FH} \cong \overline{GH}$ and $\angle F = \angle G = 90^o$



iii) No, here again $\overline{EH}$ is not the bisector of $\angle FEG$.



Q7: Find the values of x:

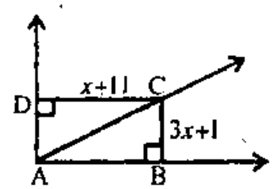
Solution:

i) Since $\angle B = \angle D = 90^o$

So $\overline{DC} = \overline{BC} \Rightarrow 3x + 1 = x + 11$

$\Rightarrow 3x - x = 11 - 1$

$\Rightarrow 2x = 10 \Rightarrow \boxed{x = 5}$



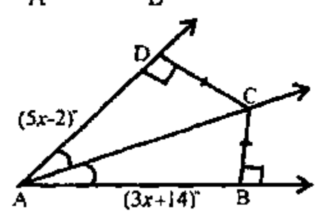
ii) Since $\overline{AC}$ is the angle bisector of $\angle BAD$

So $\angle BAC = \angle CAD$

$\Rightarrow 3x + 14 = 5x - 2$

$\Rightarrow 3x - 5x = -2 - 14$

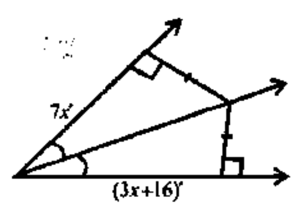
$\cancel{-}2x = \cancel{-}16 \Rightarrow x = \frac{16}{2} \Rightarrow x = 8$



iii) Here $7x = 3x + 16$

$7x - 3x = 16$

$4x = 16 \Rightarrow x = \frac{16}{4} = 2$

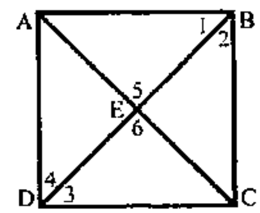


Q8: Prove that the diagonals of a square are the right bisectors of each other.

Given:

ABCD is a square

in which $\overline{AC}$ and $\overline{BD}$ are diagonals.



To Prove:

$\overline{AC}$ and $\overline{BD}$ are the right bisectors.

Proof:

Statement	Reasons
In ΔABD, $\angle 1 \cong \angle 4$	Angle opposite congruent sides
$\angle 2 \cong \angle 4$	Alternate angles
$\therefore$ $\angle 1 \cong \angle 2$	
In $\Delta AEB \longleftrightarrow \Delta BEC$	
$\overline{AB} \cong \overline{BC}$	All sides of square are equal
$\overline{BE} \cong \overline{BE}$	Common
$\angle 1 \cong \angle 2$	Proved
$\therefore$ $\Delta AEB \cong \Delta BEC$	(S.A.S)
Hence $\overline{AE} \cong \overline{EC}$	Corresponding sides
$\therefore$ $\angle BEA + \angle BEC = 180^o$	Supplementary angles
$\angle BEA \cong \angle BEC = 90^o$	
Hence $\overline{AB} \perp \overline{BD}$ and $\overline{CE} \perp \overline{BD}$.	

REVIEW EXERCISE 12

Q1: Select the correct answer and write the corresponding letter a, b, c or d in the box.

i) Which of the following are concurrent?
(a) Angles bisectors of a triangle
(b) Perpendicular bisectors of the sides of a triangle
(c) Medians of a triangle ✓ **(d) All of these**

ii) Which of the following sometimes is inside a triangle, sometimes coincides with a side of a triangle, and sometimes falls outside of a triangle?
(a) The base ✓ **(b) The altitude**
(c) The median (d) The angle bisector

iii) Perpendicular bisectors of a triangle are:
(a) Congruent ✓ **(b) Concurrent**
(c) Parallel to each other (d) Perpendicular to each other

iv) In which triangle the perpendicular bisector of the base passes through its vertex angle?
(a) Right angled (b) Scalene
✓ **(c) Isosceles** (d) Acute-angled

v) In $\triangle ABC$, medians AD, BE, and CF intersect at G. If $CF = 24$, what is the length of FG?
✓ **(a) 8** (b) 12 (c) 10 (d) 16

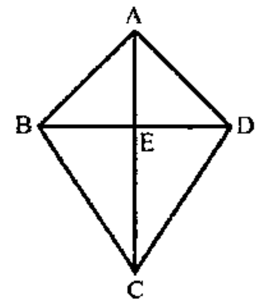
vi) The angle bisectors of a triangle meet at a point which is equidistant from............. of the triangle.
(a) The vertices ✓ **(b) The sides**
(c) Midpoints of the sides (d) All of these

vii) In an equilateral triangle, all the perpendicular bisector are:
(a) Congruent ✓ **(b) Concurrent**
(c) The angle bisector as well (d) Parallel

viii) Point of intersection of the angle bisectors of a triangle is equidistant from........... of the triangle.
(a) The vertices ✓ **(b) The sides**
(c) Midpoints of the sides (d) All of the above

Q2: Prove that if both pairs of opposite sides of a quadrilateral are congruent, then the in the given figure $\overline{AB} \cong \overline{AD}$ and $\overline{BC} \cong \overline{DC}$. Prove that $\overline{AC} \perp \overline{BD}$ and $\overline{BE} \cong \overline{DE}$. y are also parallel.

Solution:



Given $\overline{AB} \cong \overline{AD}$

And $\overline{BC} \cong \overline{DC}$

To Prove:

i) $\overline{BE} \cong \overline{DE}$

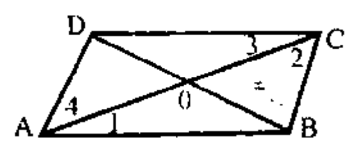
ii) $\overline{AC} \perp \overline{BD}$

Proof:

Statement	Reasons
In $\Delta ABE \longleftrightarrow \Delta ADE$	
$\overline{AB} \cong \overline{AD}$	Given
$\overline{AE} \cong \overline{AE}$	Common
$m\angle AEB \cong m\angle AED$	Opposite angles of $\overline{AB}$ and $\overline{AD}$
$\therefore$ $\Delta ABE \cong \Delta ADE$	S.A.S
Hence $\overline{BE} \cong \overline{DE}$	Corresponding sides of congruent triangles
But $m\angle AEB \cong m\angle AED$	Corresponding sides of congruent triangles
$m\angle 3 \cong m\angle 4$	
But $m\angle 3 + m\angle 4 = 180^o$	Supplementary angles
Hence $m\angle 3 = m\angle 4 = 90^o$	
$\therefore$ $\overline{AC}$ is perpendicular to $\overline{BD}$	

Q3: Prove that the diagonals of a rhombus are the right bisectors of each other.

Solution:



Given $\overline{AC}$ and $\overline{BC}$ are the diagonals of rhombus ABCD.

To Prove: $\overline{AC}$ and $\overline{BD}$ bisect each other at right angles.

Proof:

Statement	Reasons
In $\Delta ABC \longleftrightarrow \Delta BAD$	
$\overline{AB} \cong \overline{AB}$	Common
$m\angle ABC \cong \angle BAD$	Each is of opposite side of rhombus
$BC \cong AD$	S.A.S
$\Delta ABC \cong \Delta BAD$	
$\overline{AC} \cong \overline{BD}$	Corresponding sides of congruent triangles
$\overline{AB} \cong \overline{BC}$	

Q4: Prove that bisectors of the base angles of an isosceles triangle intersect each other at the right bisector of the base.

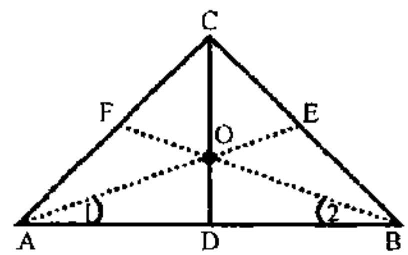
Solution:

Given in ΔABC,

$m\angle A \cong m\angle B$

To Prove:

$\overline{OD}$ is the right bisector of $\overline{AB}$ or $\overline{OD} \perp \overline{AB}$.



Proof:

Statement	Reasons
In $\Delta AOD \longleftrightarrow \Delta BOD$	O is point of intersection of the bisectors of $\angle A$ and $\angle B$.
$\overline{AO} \cong \overline{BO}$	
$m\angle 1 \cong m\angle 2$	Bisectors of two congruent angles
$\therefore$ $\Delta AOD \cong \Delta BOD$	S.A.S
Hence $m\angle 3 \cong m\angle 4$	Corresponding angles
But $m\angle 3 + m\angle 4 = 180^{o}$	Supplementary angles
$\Rightarrow$ $m\angle 3 = 90^{o}$	Hence $OD \perp \overline{AB}$

***

Additional MCQs of Unit 12:
Line Bisectors and Angle Bisectors

1. Any point on..........of a lien segment is equidistant from its end points..........
(a) Angle bisector (b) Median (c) Right bisector (d) none
✓ **Ans. (c) Right bisector**

2. The right bisectors of the sides of a triangle are..........
(a) Parallel (b) Concurrent (c) Perpendicular (d) none
✓ **Ans. (b) Concurrent**

3. The bisectors of the..........of a triangle are concurrent.
(a) Angles (b) Sides (c) Vertices (d) none
✓ **Ans. (a) Angles**

4. Bisector is a line that divides a segment into..........congruent parts.
(a) One (b) Two (c) Three (d) Four
✓ **Ans. (b) Two**

5. The point where a bisector intersects a segment is the..........of the segment.
(a) End point (b) Center point (c) Midpoint (d) none
✓ **Ans. (c) Midpoint**

6. A bisector which is perpendicular to a segment is called..........
(a) Median (b) Right bisector (c) Altitude (d) none
✓ **Ans. (a) Right bisector**

7. An angle bisector is a line which divides..........into two congruent parts.
(a) Sides (b) Triangles (c) An angle (d) none
✓ **Ans. (c) An angle**

8. A line or ray which is perpendicular to the side of a triangle is called..........
(a) Altitude (b) Bisector (c) Median (d) none
✓ **Ans. (a) Altitude**

9. The point of concurrency of the three angle bisectors of a triangle is called.........
(a) Centroide (b) The incentre (c) Bisector (d) none
✓ **Ans. (b) The incentre**

10. The incentre is..........from the three sides of the triangle.
(a) Near (b) At different distance (c) Equidistant (d) none
✓ **Ans. (c) At different distance**